ANDERSON SERANGOON JUNIOR COLLEGE

MATHEMATICS **9758**

H2 Mathematics Paper 1 (100 marks)

14 Sept 2020

3 hours

Additional Material(s): List of Formulae (MF26)

CANDIDATE NAME

CLASS [ ][ ] / [ ][ ]

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
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11	
Total	

This document consists of 18 printed pages and 2 blank pages.

1 A convergent geometric progression G has nth term denoted by u_n. It has a positive first term and common ratio r. Another geometric progression H of positive numbers has nth term denoted by v_n and common ratio $\frac{1}{R}$. If $0 < r < R < 1$, show that a new sequence whose nth term is $\ln(u_n v_n)$ is an arithmetic progression. [2]

Determine if this sequence is decreasing or increasing. Justify your answer. [2]

2 It is given that $\mathrm{f}(x) = 2kx^3 + (5k-2)x^2 + (k-5)x + 3 - 2k$, where k is a real constant. The curve with equation $y = \mathrm{f}(x)$ has only 1 x-intercept at $x = \frac{1}{2}$.

(i) Find the exact range of values of k. [3]

(ii) What can be said about the other two solutions to the equation $\mathrm{f}(x) = 0$? [1]

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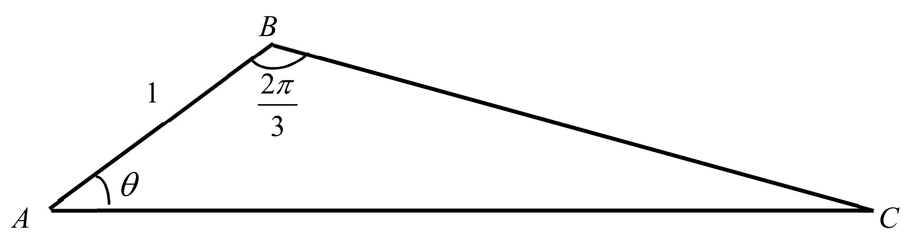


In the triangle ABC, $AB = 1$, angle $BAC = \theta$ radians and angle $ABC = \frac{2\pi}{3}$ radians (see diagram).

(i) Show that $AC = \dfrac{\sqrt{3}}{\sqrt{3}\cos\theta - \sin\theta}$. [3]

(ii) Given that θ is a sufficiently small angle, show that

$$AC \approx 1 + a\theta + b\theta^2,$$

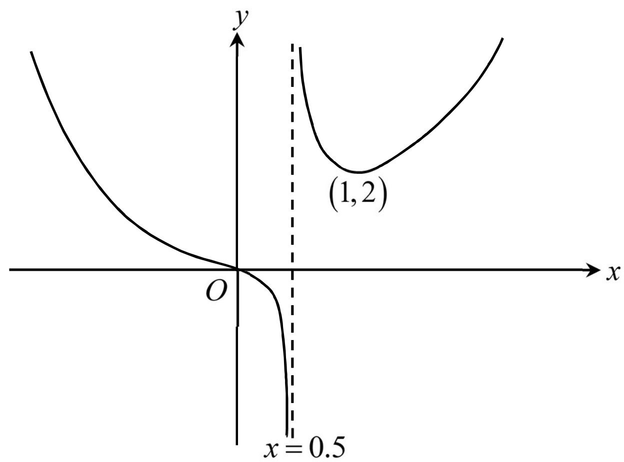
for constants a and b to be determined exactly. [3]

4 **(a)** The curve C has equation $y=\dfrac{x^2-5x+7}{4-2x}$.

By expressing the equation of C in the form $y=A\left[(3-x)+\dfrac{1}{(3-x)+B}\right]$,

where A and B are constants to be found, find a sequence of transformations which transforms the graph of C on to the graph of $y=x+\dfrac{1}{x-1}$. [4]

(b)



The diagram shows the graph of $y=\mathrm{f}(x)$ with an asymptote $x=0.5$. The curve passes through the origin O and has only one stationary point at $(1,2)$.

Sketch the graph of $y=\dfrac{1}{\mathrm{f}(x)}$, showing clearly the equations of any asymptotes and the coordinates of any turning points and x–intercepts. [3]

5 **(i)** Express $\dfrac{4}{4r^2+16r+15}$ as $\dfrac{A}{2r+3}+\dfrac{B}{2r+5}$, where A and B are constants to be determined. [1]

The sum $\displaystyle\sum_{r=1}^{n}\frac{4}{4r^2+16r+15}$ is denoted by S_n.

(ii) Find an expression for S_n in terms of n. [3]

(iii) Find the smallest value of n for which S_n is within 10^{-3} of the sum to infinity. [3]

(iv) Using the result in **(ii)**, find $\displaystyle\sum_{r=10}^{2n}\frac{2}{4r^2-1}$ in terms of n. [3]

6 Referred to the origin O, points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Point C lies on BA produced such that $BA : AC = \lambda : \mu$, where $\lambda, \mu > 0$.

(i) Show that $\overrightarrow{OC}$ is given by $\dfrac{\lambda+\mu}{\lambda}\mathbf{a} - \dfrac{\mu}{\lambda}\mathbf{b}$. [1]

(ii) It is given that $\mathbf{a}$ is a unit vector, $|\mathbf{b}| = 3$ and angle AOB is $\cos^{-1}\left(\dfrac{2}{3}\right)$. Find the ratio $BA : AC$ such that C, which is on the line AB, is nearest to O. [4]

The position vector $\mathbf{v}_k$ is such that $\mathbf{v}_k = \mathbf{a} + k(\mathbf{b} - \mathbf{a})$ where k is a positive integer. If $\overrightarrow{OS} = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 + \ldots + \mathbf{v}_n$,

(iii) find $\overrightarrow{OS}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and n. Simplify your answer. [2]

(iv) If $\lambda = 2$ and $\mu = 3$, find the area of triangle OCS in the form $k|\mathbf{a} \times \mathbf{b}|$, where k is in terms of n. [3]

7 **(a)** The function f is such that $\mathrm{f} : x \mapsto -2x^2 + 4x + 1, \quad x \in \mathbb{R}, x \le a$.

State the greatest value of a such that f^{-1} exists and for this value of a, find f^{-1} in a similar form. [4]

(b) The function g is a strictly decreasing and continuous function such that $\mathrm{g}(x) = -\mathrm{g}(-x)$ for $x \in \mathbb{R}$. The coordinates of certain points on the curve of $y = \mathrm{g}(x)$ are as follows:

x	-6	-5.7	-5	-4	-3.2	-3	-1	0	2	4.3
y	7	$4\sqrt{3}$	6	$3\sqrt{3}$	4.3	4	$2\sqrt{3}$	0	-3.7	-5.5

(i) State the value of $\mathrm{g}^{-1}(-6)$. [1]

Another function h is defined by $\mathrm{h} : x \mapsto 4\sin 2x$ for $0 \le x \le \dfrac{\pi}{2}$.

(ii) Find the exact range of the composite function gh. Find the range of values of x such that the composite function gh satisfies the inequality $\left|\mathrm{g}^{-1}\mathrm{h}(x) + 3\right| < 2$. Leave your answer in exact form. [4]

7 **(c)** The function k is defined by

$$\mathrm{k}: n \mapsto \begin{cases} \sqrt{n} + \dfrac{n}{2}, & \text{for odd positive integer } n \\ n, & \text{for even positive integer } n. \end{cases}$$

Explain clearly whether the composite function k^2 exists. [1]

8 **Do not use a calculator in answering this question.**

(a)(i) Solve the equation $w^2 = 3 - 4\mathrm{i}$, giving your answers in cartesian form $a + \mathrm{i}b$. [4]

(ii) Hence find the roots of the equation $z^2 - 4\mathrm{i}z + 4\mathrm{i} - 7 = 0$, giving your answers in cartesian form $p + q\mathrm{i}$. [2]

(b) For positive integer n, a complex number z is such that $|z|^n = \dfrac{1}{\sqrt{2}}$. By considering the conjugate of $1 + 2z^{2n}$, show that $\dfrac{2z^n}{1+2z^{2n}}$ is a real number. [4]

9 **(a)(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\sin(x^3+2)\right]$. [1]

(ii) Find $\int 2x^5 \cos(x^3+2)\,\mathrm{d}x$. [2]

(b)(i) Using an algebraic approach, solve the inequality $\dfrac{x-16}{x^2-16} \leq 1$. [3]

(ii) Find $\int \left(\dfrac{x-16}{x^2-16} - 1\right) \mathrm{d}x$. [3]

(iii) Hence find the exact value of $\int_{-1}^{1} \left|\dfrac{x-16}{x^2-16} - 1\right| \mathrm{d}x$ in the form of $\ln\dfrac{a}{b}$, where a and b are real constants. [3]

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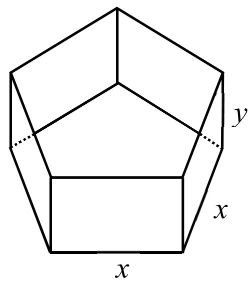

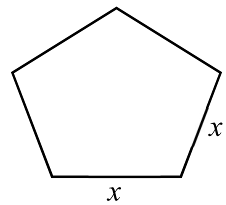

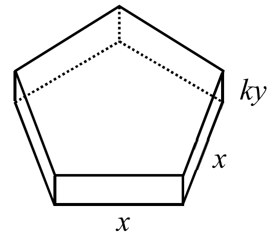

Crayole, a handicraft company, requires a container of negligible thickness to hold 200 cm^3 of party slime. The cross-section of the container is a regular pentagon with sides x cm and the height of the container is y cm. The sides of its lid is x cm and has a depth ky cm, where $0 < k \leq 1$ (see diagram).

It is known that the area of a regular pentagon with sides x cm is given by $\dfrac{ax^2}{4}$, where a is a positive fixed constant.

(i) Use differentiation to find, in terms of a and k, the exact value of x which gives a minimum total external surface area of the container and the lid. [6]

(ii) Find the ratio $\dfrac{y}{x}$ in terms of a and k in this case, simplifying your answer. [2]

(iii) Find the range of values of $\dfrac{y}{x}$ in terms of a. [2]

(iv) Find the exact value of k, in terms of a, if the company requires the sides of the container to be $\dfrac{3}{4}$ of its height. [2]

11 **(a)** At the beginning of January 2010, Mr Wong's fish farm has 40 000 fish breeding in a large pond. In the period between January and December each year, the number of fish increases by 9%. At the end of December each year, 5400 fish are harvested and sold at the fish markets for \$6 per fish. No fish died or are being poached while he is in business.

(i) Find the number of fish just after the nth harvest, giving your answer in the form $A - B(1.09)^n$, where A and B are constants to be determined. [3]

(ii) Find the earliest month and year in which there will be no more fish in the pond. [2]

Mr Wong has been thinking of retiring since January 2010. Every December just after the harvest, a nearby fish farm offers to buy over Mr Wong's business by paying \$8 per fish for his remaining stock.

(iii) If he wishes to retire by selling off his fish farm business with the maximum gain after the nth harvest, find the value of n. [3]

(b) White-tailed deer can be found in forests of Southern Canada. A wildlife biologist is investigating the change of a population of white-tailed deer of size x (thousands) at time t (years). He observed that on average, their birth rate is 4000 per year and their death rate is proportional to the square of the population present.

(i) When the population was 5000, the rate of increase of the population at that instant was 1750 per year. Assuming that x and t are continuous variables, form a differential equation relating x and t. [2]

(ii) There were 2000 deer in the forest initially. Solve this differential equation to obtain x as an exact function of t. [4]

End of Paper

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