



**NATIONAL JUNIOR COLLEGE**  
**SENIOR HIGH 2**  
**Higher 2**

NAME

CLASS

2ma2

REGISTRATION  
NUMBER

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**MATHEMATICS**

**9758**

**Preliminary Examination**

**12 September 2022**

**Paper 2**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and registration number in the boxes above.  
Please write clearly and use capital letters.

Write in dark blue or black pen.  
You may use an HB pencil for any diagrams or graphs.  
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.  
Write your answers in the spaces provided in the question paper.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
The use of an approved graphing and/or scientific calculator is expected, where appropriate.  
All relevant working, statements and reasons must be shown in order to obtain full credit for your solution.

You are reminded of the need for clear presentation in your answers.  
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in the brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 100.

Question Number	Marks Possible	Marks Obtained
1	7	
2	9	
3	13	
4	11	
5	3	
6	10	
7	7	
8	7	
9	10	
10	11	
11	12	
Presentation Deduction		- 1 / - 2
<b>TOTAL</b>	<b>100</b>	

## Section A: Pure Mathematics [40 marks]

1 It is given that  $f'(x) = f(x) \ln(\sec ax)$  and  $f(0) = \frac{1}{2}$ , where  $a$  is a real non-zero constant.

- (i) Show that  $f''(x)f(x) = [f'(x)]^2 + a[f(x)]^2 \tan ax$ . Hence, find the Maclaurin series of  $f(x)$  in terms of  $a$ , up to and including the term in  $x^3$ . [4]

Assume for the remainder of this question that  $a = 3$ .

The function  $g(x)$  is an anti-derivative of  $f(x)[1 + \ln(\cos 3x)]$ , such that  $g(0) = 0$ .

- (ii) Deduce the Maclaurin series of  $g(x)$  up to and including the term in  $x^3$ . [3]

2 The function  $f$  is defined by  $f(x) = \begin{cases} 3x^2 + 2, & \text{for } x \in \mathbb{R}, 0 < x \leq 1, \\ \frac{1}{x}, & \text{for } x \in \mathbb{R}, x > 1. \end{cases}$

- (i) Sketch the graph of  $y = f(x)$ . [2]

- (ii) Show that  $f^2$  exists and define  $f^2$  in a similar form. [4]

- (iii) Find  $f^{-1}$  in a form similar to  $f$ . [3]

3 A curve  $C$  has equation  $y^2 - x^2 = 9$ .

- (i) Sketch  $C$ , labelling clearly the coordinates of the turning points, the equations of the asymptotes, and the angle between the asymptotes. [3]

- (ii) Show that  $\frac{d}{d\theta}(\sec \theta \tan \theta) = 2 \sec^3 \theta - \sec \theta$ . [2]

The region bounded by  $C$ , the positive  $x$ - and  $y$ -axes, and the line  $x = 4$  is denoted by  $S$ .

- (iii) Use the substitution  $x = 3 \tan \theta$  and the result in part (ii) to show that the exact area of  $S$  can be expressed in the form  $p + q \ln r$  for some rational constants  $p$ ,  $q$  and  $r$  to be determined. [6]

- (iv) Without using integration, find the exact value of  $\int_3^5 \sqrt{x^2 - 9} \, dx$  in a form similar to your answer in part (iii). [2]

- 4 A curve  $C$  has equation

$$y = \frac{a^2}{x} - \frac{1}{x-k},$$

where  $a$  and  $k$  are positive constants, with  $a > 1$ .

- (i) Find the  $x$ -coordinates of the stationary points of  $C$  in terms of  $a$  and  $k$ . [3]

It is given that the point  $P$  is the stationary point of  $C$  with the larger  $x$ -coordinate, and  $L$  is the locus of  $P$  as  $a$  varies.

- (ii) Find a cartesian equation of  $L$  as  $a$  varies, expressing  $y$  in terms of  $x$  and  $k$ . [4]

Assume now that  $k = 4$ . It is further given that  $Q$  has the same equation as  $C$  with  $a = 3$ .

- (iii) Sketch  $Q$  and  $L$  on the same diagram, for  $x > 4$ . Label clearly the coordinates of any turning points and any points where the curves cross the  $x$ -axis, as well as the equations of any asymptotes. [4]

### Section B: Probability and Statistics [60 marks]

- 5 The random variable  $W$  has the distribution  $B(n, p)$  and a mode  $m$ . By considering the inequality  $P(W = m) \geq P(W = m + 1)$ , show that

$$m \geq E(W) + p - 1. \quad [3]$$

- 6 Every weekday morning, Jo arrives at the bus stop nearest to her home to catch bus 156 to Bishan MRT station. The waiting time for bus 156 follows a normal distribution with mean 9 minutes and standard deviation 1.5 minutes, and the time taken for the journey from the bus stop to Bishan MRT station follows a normal distribution with mean 10 minutes and standard deviation 1.1 minutes.

- (i) Find the probability that the mean waiting time for bus 156 on 50 randomly chosen weekdays exceeds 9.3 minutes. [3]
- (ii) Explain why your calculations in part (i) would still be valid even if the waiting time for bus 156 on a weekday is not known to be normally distributed. [2]
- (iii) If the train at Bishan MRT station leaves at 7.00am, find the latest time (correct to the nearest minute) that Jo has to reach the bus stop if she misses the train less than 30% of the time. [2]
- (iv) In the following year, the waiting time for bus 156 is increased by 30% due to COVID-19 infections among bus drivers. Find the probability that the waiting time for a randomly chosen bus 156 in the following year differs from the original waiting time for a randomly chosen bus 156 by at most 2 minutes. [3]

- 7 On average, 7.8% of markers produced by a factory are faulty. Each day, the factory's quality manager picks a random sample of  $n$  markers and inspects the number of faulty markers found in the sample. The number of faulty markers found in the sample is the random variable  $X$  and whether the markers are faulty are independent of one another.

- (i) Explain what is meant by a random sample in this context. [1]
- (ii) Given that the probability that more than  $(n - 4)$  markers in the random sample are found to be non-faulty is less than 0.3, find the least possible value of  $n$ . [2]

Assume for the remainder of this question that  $n = 10$ .

The samples in 50 randomly chosen days are collected and the number of faulty markers in each sample is recorded.

- (iii) Find the probability that exactly 35 of the samples each contains at least one faulty marker, given that none of the 50 samples contain more than 2 faulty markers each. [4]

- 8 The table shows the probability distribution of a discrete random variable  $Y$ .

$y$	-1	0	1	2	3
$P(Y = y)$	$\frac{32}{81}$	$a$	$\frac{8}{27}$	$b$	$c$

- (i) Given that  $E(|Y - 3|) = \frac{232}{81}$ , show that  $2b + 3c = \frac{19}{81}$ . [3]
- (ii) Given further that  $\text{Var}(Y) = \frac{7736}{6561}$ , find the values of  $a$ ,  $b$  and  $c$  in rational form. [3]
- (iii) Justify if  $Y$  follows a binomial distribution. [1]

- 9 Osoi is frequently late for school. His mean travelling time to school is 24.5 minutes. The Year Head asks Osoi to take an alternative route to school and to record his travelling time,  $t$  minutes, from home to school every morning on the alternative route. The results for 72 randomly chosen mornings when Osoi took the alternative route are summarised as follows:

$$\sum(t - 20) = 215; \quad \sum(t - 20)^2 = 3234$$

- (i) Calculate unbiased estimates of the population mean and variance of the travelling time for the alternative route. [2]
- (ii) Using a 5% level of significance, test whether Osoi's mean travelling time from home to school has shortened after taking the alternative route to school. You should state your hypotheses and define any symbols that you use. [5]

Suppose the Year Head wants to investigate if the alternative route to school has made a difference to Osoi's mean travelling time in the morning instead of whether it has shortened his mean travelling time in the morning, at the same level of significance.

- (iii) Find the critical region for this test. [3]

- 10** A financial magazine publishes an annual ranking of the financial services companies in the world. The ranking is based on sales, profit, assets and market value. A random sample of 7 pairs of assets, \$  $s$  billion, and profit, \$  $p$  billion, is shown in the table below.

Assets, $s$	326.7	394.5	491.9	2832.2	4159.9	4301.7	4914.7
Profit, $p$	2	2.6	3.4	17.9	31.3	39.3	65.8

- (i) Draw a scatter diagram for these values, labelling the axes clearly. [1]

It is thought that the profit  $p$  can be modelled by one of the following models.

$$\text{Model I: } p = as + b \text{ or Model II: } p = as^2 + b,$$

where  $a$  and  $b$  are constants.

- (ii) Find the value of the product moment correlation coefficient between
- (a)  $s$  and  $p$ , [1]
- (b)  $s^2$  and  $p$ . [1]
- (iii) Use your answers to parts (i) and (ii) to explain why Model II is a better model than Model I. [2]
- (iv) Find the equation of the regression line of  $p$  on  $s^2$ . [1]
- (v) Using the regression line in part (iv), estimate the value of  $p$  if  $s = 300$ . Comment on the reliability of the estimate you have obtained. [2]
- (vi) The original data set contains 8 pairs of data with the regression line  $p = 0.010423445s - 4.043538369$ . Find the value of  $p$  for the missing pair if the corresponding value of  $s$  is 3689. [3]

- 11** During Senior High Orientation, one of the bonding games to be played is Avalon. In each round of this game, a student is randomly selected within the Orientation Group to be the “Avalon”. The student chosen to be the “Avalon” wins if the group is unsuccessful in guessing who the Avalon is within a given number of attempts.

In a particular Orientation Group, there are a total of 5 Orientation Group Leaders (OGLs), consisting of 2 boys and 3 girls, and 16 orientees, consisting of 6 boys and 10 girls. The OGLs appoints one particular girl from the 16 orientees to be the Chairperson of the Orientation Group.

The Orientation Group plays a total of five rounds of Avalon. In each of the first two rounds, the Avalon is randomly selected from the group of 5 OGLs. In each of the subsequent three rounds, the Avalon is randomly selected from the group of 16 orientees. It is possible for the same person to be randomly selected to be the Avalon in multiple rounds of the game.

- (i) Find the probability that in the five rounds, the Avalons selected are all distinct. [2]
- (ii) Find the probability that in the five rounds, a girl is selected in the first round and a boy is selected in the second round. [2]
- (iii) For the five rounds, events  $A$  and  $B$  are defined as follows.

- $A$ : The same OGL is selected in both of the first two rounds.  
 $B$ : Exactly two orientees are selected in the last three rounds.

Find the probability that  $A$  or  $B$  occurs, but not both. [4]

- (iv) Find the probability that in the five rounds, the Chairperson of the Orientation Group is selected exactly once. [2]
- (v) Find the probability that in the five rounds, all the Avalons selected are girls, given that the Chairperson of the Orientation Group is selected exactly once. [2]