ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 9758/01

Paper 2 **3 September 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **5** printed pages.

Anglo-Chinese Junior College **[Turn Over**

Section A: Pure Mathematics [40 marks]

1 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$**,** where $\mathbf{a}$ and $\mathbf{b}$ are not parallel, $\mathbf{b}$ is a unit vector, and $\angle AOB = 45^\circ$. The point R has position vector given by $\mathbf{r} = 3\mathbf{a} + 5\mathbf{b}$.

Find

(i) the position vector of the point where OR meets AB, [3]

(ii) the length of projection of $\overrightarrow{OR}$ on $\overrightarrow{OB}$, leaving your answer in terms of $|\mathbf{a}|$. [3]

2 By considering $\dfrac{1}{r(r-2)} - \dfrac{1}{r(r+2)}$, show that

$$\sum_{r=3}^{n} \frac{1}{(r-2)r(r+2)} = a + \frac{b}{(n-1)(n+1)} + \frac{c}{n(n+2)},$$

where a, b and c are constants to be determined. [4]

(i) State the value of $\displaystyle\sum_{r=3}^{\infty} \frac{1}{(r-2)r(r+2)}$. [1]

(ii) Find $\displaystyle\sum_{r=5}^{n} \frac{1}{r(r+2)(r+4)}$ in terms of n. [3]

3 **(a)** An ellipse has equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where $b > a > 0$.

(i) Show that the area A of the region enclosed by the ellipse is given by

$$A = \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\ \mathrm{d}x\,. \qquad [1]$$

By using the substitution $x = a\sin\theta$, find A in terms of a, b and π. [4]

(ii) The region enclosed by the ellipse is rotated about the x-axis through π radians to form an ellipsoid. Find the volume of the ellipsoid formed, in terms of a, b and π. [3]

(b) When a continuous function, $y = \mathrm{f}(x)$, $a \le x \le b$, is rotated completely about the x-axis, the resulting curved surface area is given by

$$S = \int_a^b 2\pi y\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x\,.$$

By considering the circle with equation $x^2 + y^2 = r^2$ where $-r \le x \le r$, show that the surface area of a sphere with radius r is $4\pi r^2$. [4]

4 The folium of Descartes is an algebraic curve defined by the equation

$$x^3 + y^3 = 3axy,$$

where a is a real constant.
Consider the curve when $a = 1$.
Show that by setting $y = xt$, where t is a parameter such that $t \in \mathbb{R}$,

$$x = \frac{3t}{1+t^3}, \quad t \neq k,$$

where k is a real number to be determined.
Hence write down the expression for y in terms of t. [3]

(i) Show that

$$x + y = \frac{3t}{t^2 - t + 1},$$

and, by considering limits or otherwise, find the equation of the oblique asymptote of the curve. [3]

(ii) Sketch the graph of the folium of Descartes when $a = 1$, showing clearly the coordinates of any intersection with the axes, and the equation of any asymptote(s). [2]

(iii) Find the equation of the tangent to the curve at $t = 2$, and determine if the tangent cuts the curve again, giving the coordinates of the intersection if it does. [6]

Section B: Probability and Statistics [60 marks]

5 A school tennis team comprises of 6 boys and 6 girls.

(i) Given that the members of the tennis team all have different heights, find the number of ways a group of 4 students of increasing heights can be chosen from the team. [1]

(ii) A delegation of at least one student from the team is to be chosen to attend a convention. Find the number of ways this delegation can be chosen. [2]

(iii) Andy, a boy, and Beth, a girl, are members of the tennis team. A group of 8 students from the team is to be chosen to attend a dinner, seated at a round table.
Find the probability that the boys and the girls in the group alternate such that the group includes Andy and Beth who are seated next to each other. [2]

6 A class proposes the following game for their school's fundraising event.
Four discs numbered '1', '3', '5' and '7' respectively are placed in Bag A. Another four discs numbered '2', '4', '6', and '8' respectively are placed in Bag B. A player chooses two discs at random and without replacement from each of the two bags. It is assumed that selections from the two bags are independent of each other.
Let X represent the difference between the numbers on the two discs drawn from Bag A, and Y represent the difference between the numbers on the two discs drawn from Bag B. The player wins \$$X$ if $X = Y$. Otherwise, the player wins nothing.

(i) Show that $\mathrm{P}(X = 2) = \frac{1}{2}$ and find the probability distribution of X. [3]

(ii) Find the probability that the player wins a prize. [2]

The game is to be played using a \$$k$ coupon, where k is a positive integer. Find the minimum value of k in order for the class to earn a profit for each game, justifying your answer. [2]

7 A concert promoter claims that the mean price of a ticket to a pop concert is \$200. A media company collected information on ticket price, \$$x$, for 50 randomly chosen people who bought pop concert tickets. The results are summarised as follows.

$$\sum(x-200) = 450 \qquad \sum(x-200)^2 = 55\ 000$$

(i) Test, at the 4% level of significance, the concert promoter's claim that the mean price of a ticket to a pop concert is \$200. You should state your hypotheses and define any symbols you use. [5]

(ii) The media company took another random sample of n tickets, and found that the average ticket price for this sample is \$206. If the standard deviation of ticket price is now known to be \$32.25, find the maximum value of n such that there is insufficient evidence at the 4% level of significance to reject the concert promoter's claim. [2]

8 A company that organizes live concerts believes that the popularity of an artiste affects his/her concert ticket sales. The popularity of an artiste can be measured by an index, x, such that $1 \le x \le 10$, where 10 indicates most popular and 1 indicates least popular. A study was conducted over six months to investigate the relationship between the popularity index of eight artistes and their concert ticket sales. The results are summarised in the following table.

Artiste	A	B	C	D	E	F	G	H
Popularity Index, x	1.2	2.0	2.7	3.8	4.8	5.6	6.9	8.0
Concert Ticket Sales (hundreds of thousands), \$$y$	2.2	4.5	5.8	7.3	7.4	9.0	9.9	10.8

(i) Draw a scatter diagram for these values, labelling the axes. [1]

(ii) It is thought that concert ticket sales y can be modelled by one of the formulae

$$y = ax^2 + b \qquad \text{or} \qquad y = c\ln x + d,$$

where a, b, c and d are positive constants.
Use your diagram in **(i)** to explain which of the two is a more appropriate model, and calculate its product moment correlation coefficient, correct to 4 decimal places. [2]

(iii) The data for a particular artiste appears to be recorded wrongly. Indicate the corresponding point on your diagram by labeling it P. Find the equation of the least squares regression line for the remaining points using the model that you have chosen in **(ii)**. [2]

(iv) The ticket sales for a new artiste is found to be \$800, 000. Estimate the popularity index of this artiste and comment on the reliability of your estimate. Explain why neither the regression line of $\ln x$ on y nor x^2 on y should be used. [3]

9 The teacher in-charge of the Harmonica Ensemble observed that a pair of twins often turned up late for practice. Attendance records show that the older twin, Albert, is late for practice 65% of the time. When the younger twin, Benny, is late for practice, Albert is also late 97.5% of the time. When Benny is not late for practice, Albert is late 56.875% of the time.

(i) Show that the probability that Benny is late for a practice is 0.2. [3]

(ii) Find the probability that only one of the twins will be late for the next practice. [2]

The teacher observed that another student, Carl, is late for practice 50% of the time. The probability that all three will be late for practice is 0.098. Given that the event of either twin being late for practice is independent of the event that Carl is late for practice, find the probability that neither the twins nor Carl is late for a practice. [3]

10 In this question you should state the parameters of any distributions you use.
Mary runs a noodle stall by herself. The noodles are prepared to order, so she prepares the noodles only after each order, and will only take the next customer's order after the previous customer is served his noodles.
The time taken for a customer to place an order at the stall follows a normal distribution with mean 60 s and standard deviation σ s. The time taken for Mary to prepare and serve a customer's order also follows a normal distribution, with mean 300 s and standard deviation 50 s.
The probability that a customer takes not more than 40 s to place an order is 0.16. Show that $\sigma = 20.11$, correct to 2 decimal places. [2]

(i) Let A be the probability that a randomly chosen customer takes between 57 s and 63 s to place an order with Mary, and B be the probability that a randomly chosen customer takes between 49 s and 55 s to place an order with Mary. Without calculating A and B, explain, with the aid of a diagram, how A and B compare with each other. [2]

(ii) There is a 0.1% chance that a randomly chosen customer has to wait for more than k seconds for his noodles to be served after placing his order. Find k. [1]

(iii) A man visits the noodle stall on 10 separate occasions. Find the probability that his average waiting time per visit after placing his order is less than 4.5 minutes. [2]

(iv) In an effort to shorten wait time, Mary improves the ordering and cooking processes such that the time taken for a customer to place an order is reduced by 5%, while the time taken to prepare and serve a customer his noodles is reduced by 10%. Find the largest number of customers she can serve in 1 hour for at least 80% of the time. State one assumption you made in your calculations. [5]

11 A factory manufactures a large number of wine glasses. It is found that on average, a proportion p of the wine glasses are chipped. A distributor purchases batches of wine glasses from the factory, each batch consisting of n wine glasses. A batch of wine glasses is rejected if it has more than 1 chipped wine glass. Let X be the number of wine glasses that are chipped in one batch. State two assumptions for X to be well-modelled by a binomial distribution. [2]
Show that A, the probability that one batch of wine glasses will not be rejected, is given by the formula

$$A = (1-p)^{n-1}\left[1+(n-1)p\right]. \qquad [2]$$

(a) Given that $p = 0.02$, and the probability that a batch of wine glasses is not rejected is at least 0.9, find the largest possible value of n. [2]

(b) The distributor also purchases wine glasses from another factory in batches of 40 and it is found that $A = 0.73131$.

(i) Find p correct to 5 decimal places. [1]

(ii) The mean and standard deviation of X are denoted by μ and σ respectively. Find $\mathrm{P}(\mu - \sigma < X < \mu + \sigma)$. [2]

(iii) The distributor has a one-year contract with this factory such that every week, the factory produces 20 batches of wine glasses. According to the contract, the distributor will receive a compensation of \$100 for each batch of wine glasses it rejects every week. Assuming that there are 52 weeks in a year, find the probability that the total compensation in the one-year contract period is more than \$30 000. [4]