TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME	
CIVICS GROUP	

H2 MATHEMATICS **9758/01**

Paper 1 **16 September 2020**

3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use	
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This document consists of 27 printed pages and 1 blank page.

1 Without using a calculator, find the range of values of x that satisfies the inequality $\dfrac{2x^3-3x^2+x+4}{x+3}>1.$ [4]

2 A sequence $u_1, u_2, u_3, \cdots$ is such that

$$u_n=\frac{1}{n^3} \quad \text{and} \quad u_{n+1}=u_{n-1}-\frac{6n^2+2}{\left(n^2-1\right)^3}.$$

(i) By considering $\sum_{n=2}^{N}\left(u_{n-1}-u_{n+1}\right)$, find $\sum_{n=2}^{N}\frac{3n^2+1}{\left(n^2-1\right)^3}$. [3]

(ii) Give a reason why the series in part **(i)** is convergent and state the sum to infinity. [2]

3 The complex number z is given by $z=r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $0\le\theta\le\frac{\pi}{2}$.

Given that $w=-1+\sqrt{3}\mathrm{i}$, find the exact value of θ such that $\dfrac{w^2}{(z^*)^3}$ is a real number. [5]

4 **(i)** Prove that $\cos\theta+\cos2\theta+\cos3\theta+\cos4\theta=4\cos\left(\frac{\theta}{2}\right)\cos\theta\cos\left(\frac{5\theta}{2}\right)$. [2]

A curve C has equation $y=\sin x+\frac{1}{2}\sin2x+\frac{1}{3}\sin3x+\frac{1}{4}\sin4x$, where $0\le x\le\pi$.

(ii) Using the result in part **(i)**, find in terms of π, the x-coordinates of the stationary points of C. [4]

(iii) For the stationary point where the x-coordinate is such that $0<x<\frac{\pi}{2}$, determine whether it is a maximum or minimum turning point. [2]

5 **(a)** The function f is defined by

$$\mathrm{f}: x \mapsto \frac{4}{x-2}, \quad x \in \mathbb{R}, \ x > 2.$$

(i) Show that f has an inverse. [2]

(ii) Solve exactly the equation $\mathrm{f}^{-1}(x) = x$. [3]

(b) The functions g and h are defined by

$$\mathrm{g}: x \mapsto \frac{1}{1-px}, \qquad x \in \mathbb{R}, \ x > \frac{1}{p},$$
$$\mathrm{h}: x \mapsto \ln(q-x), \quad x \in \mathbb{R}, \ x < q.$$

where p and q are positive constants.

(i) Show that the composite function hg exists. [2]

(ii) Write down an expression for hg (x) in terms of p and q. Find the range of hg in terms of q. [2]

6 **(a)** **(i)** Using standard series from the List of Formulae (MF26), expand $\ln(2+2\sin x)$, up to and including the term in x^3. [4]

(ii) Deduce the series expansion for $y = \frac{\cos x}{1+\sin x}$, up to and including the term in x^2. [2]

(b) In the triangle PQR, $PQ = 2$, $\angle PRQ$ and $\angle RPQ$ are $\frac{\pi}{6}$ and $\left(\frac{\pi}{3} - \theta\right)$ radians respectively. Given that θ is a sufficiently small angle, show that

$$QR \approx a + b\theta + c\theta^2,$$

where a, b and c are constants to be determined. [5]

7 **(a)** Give a geometrical interpretation of what it means when

(i) solving the equations $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} \bullet \mathbf{n} = k$ gives infinitely many solutions, where **a**, **b** and **n** are constant vectors, *t* is a real parameter and *k* is a constant scalar, [1]

(ii) solving the equations $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + m\mathbf{d}$ does not give a unique solution, where **a**, **b**, **c** and **d** are constant vectors, *t* and *m* are real parameters. [1]

(b) A plane *p* contains the point *A* with position vector $3\mathbf{i} + 9\mathbf{k}$ and is perpendicular to vector $-4\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$.

(i) Find the exact shortest distance from the origin *O* to plane *p*. [2]

(ii) By considering the plane containing the point *D* with coordinates $(3, -2, 4)$ and parallel to *p* or otherwise, determine whether the origin *O* and *D* are on the same or opposite side of *p*. [2]

(iii) Show that the coordinates of the foot of the perpendicular from *D* to *p* is $\left(1, \frac{1}{2}, \frac{11}{2}\right)$. [2]

(iv) Hence find an equation that describes the set of all points which are of the same distance from *p* as *D* but on the opposite side of *p*. [2]

8 **(i)** Show that the differential equation $\frac{dy}{dx} = 5(x - y)$ may be reduced by the substitution $w = x - y$ to $\frac{dw}{dx} = 1 - 5w$. Hence find the general solution for *y* in terms of *x*. [6]

(ii) Find the particular solution of the differential equation $\frac{dy}{dx} = 5(x - y)$ when $x = 0$ and $y = -1$. [2]

(iii) Sketch the graph of the particular solution found in part **(ii)**, showing clearly the behaviour of the graph as $x \to \infty$, the coordinates of the axial intercept(s) and equations of any asymptote(s). [3]

9 **(a)** A curve C has equation $y=\dfrac{(x+2)(x+5)}{x+1}$.

(i) Find, using an algebraic method, the range of values that y cannot take. [4]

(ii) Sketch the curve C, indicating clearly any equations of asymptotes and coordinates of turning points and axial intercepts. [3]

(iii) Hence find the range of values of m such that the equation $(x+2)(x+5)=(mx+m+5)(x+1)$ has 2 real roots. [2]

(b) The curve C_1 has equation $\dfrac{(x-a)^2}{b}+\dfrac{(y-c)^2}{d}=1$. The curve C_1 has centre (3, 0) and passes through the points with coordinates (3, 2) and (9, 0).

(i) Find the exact values of a, b, c, and d. [2]

(ii) Describe a sequence of transformations that maps the graph of C_2 with equation $x^2+y^2=4$ onto C_1. [2]

10 Chloe plans to invest $\$k$ on the first day of every month with the bank, starting on 1 January 2021. The bank offers two different investment plans:

Plan *A*: Each $\$k$ invested earns a fixed bonus of \$5 at the end of every month for which it has been in the account. This bonus is added to the account. The accumulated bonuses themselves do not earn any further bonus.

(a) If Chloe takes up Plan *A*, find the value of k such that the total value of all the investments, including bonuses, is worth \$22 200 at the end of 31 December 2022. [3]

Plan *B*: The interest rate is 0.2% per month, so that on the last day of each month, the amount in the account on that day is increased by 0.2%.

(b) Assume that Chloe takes up Plan *B* for the rest of the question.

(i) Find, in terms of k, the amount of interest earned from the first $\$k$ invested on 1 January 2021 at the end of 31 December 2022. [1]

(ii) Find the minimum value of k (to the nearest integer) so that the total amount in the account is worth at least \$22 200 at the end of 31 December 2022. [3]

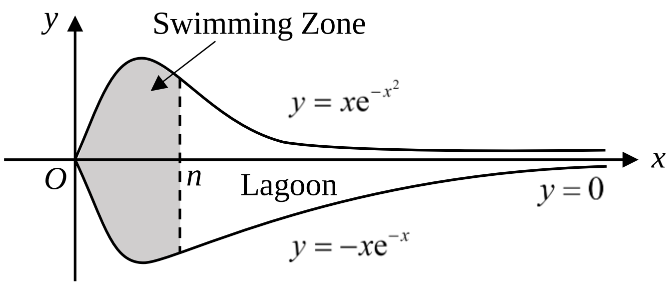
(iii) Let $k=900$. Find the month in which the total in the account will first exceed \$30 000. Explain whether this occurs on the first or last day of the month. [5]

11 The indefinite integral $\int x\mathrm{e}^{-\mathrm{f}(x)}\,\mathrm{d}x$ is denoted by I.

(i) Use the substitution $u = x^2$ to find I when $\mathrm{f}(x) = x^2$. [2]

(ii) Find I when $\mathrm{f}(x) = x$. [2]

The diagram shows a lagoon bounded by two curved banks modelled by the equations $y = x\mathrm{e}^{-x^2}$ and $y = -x\mathrm{e}^{-x}$, where $x > 0$. The safe swimming zone, as shown shaded below, is defined as the area bounded by the curves and the line $x = n$.



(iii) Show that the area of the safe swimming zone in km^2, in terms of n, is

$$\frac{3}{2} - \frac{1}{2}\mathrm{e}^{-n^2} - \mathrm{e}^{-n}(n+1).$$

Hence deduce the area of the lagoon. [4]

(iv) Given that $n = 1$, find the area of the safe swimming zone in the form $a + b\mathrm{e}^{-1}$, where a and b are constants to be determined. [2]

(v) The safe swimming zone in part **(iv)** is further divided into two parts of equal area by a straight line with equation $x = k$, where $0 < k < 1$. Find the value of k. [2]

End of Paper