YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME			
CG		INDEX NO	

MATHEMATICS 9758/01

Paper 1 **4 September 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7
Marks							

Question	8	9	10	11
Marks				

Total marks	

1 **(i)** Expand $\sin\left(\frac{\pi}{4}-2x\right)$ in ascending powers of x, up to and including the term in x^3. [3]

(ii) The first two non-zero terms found in part **(i)** are equal to the first two non-zero terms in the series expansion of $(a+bx)^{-1}$ in ascending powers of x. Find the exact values of the constants a and b. Hence find the third exact non-zero term of the series expansion of $(a+bx)^{-1}$ for these values of a and b. [3]

2 **(a)** Vectors **a** and **b** are such that $\mathbf{a} \neq 0$ **,** $\mathbf{b} \neq 0$ and $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$. Show that **a** and **b** are perpendicular. [2]

(b) Referred to the origin O, points C and D have position vectors $\mathbf{c}$ and $\mathbf{d}$ respectively. Point P lies on OC produced such that $OC:CP = 1:\lambda-1$, where $\lambda > 1$. Point M lies on DP, between D and P, such that $DM:MP = 2:3$. Write down the position vector of M in terms of λ, $\mathbf{c}$ and $\mathbf{d}$. Hence, find the area of triangle OPM in the form $k\lambda\left|\mathbf{c}\times\mathbf{d}\right|$, where k is a constant to be found. [4]

3 The function f is defined by

$$f(x)=\begin{cases}(x-2a)^2 & \text{for } 0 \le x < 2a,\\ 2ax-4a^2 & \text{for } 2a \le x < 4a,\end{cases}$$

where a is a positive real constant and that $f(x+4a)=f(x)$ for all real values of x.

(i) Sketch the graph of $y=f(x)$ for $-3a \le x \le 8a$. [3]

(ii) Hence find the value of $\int_{-2a}^{8a} f(|x|)\,dx$ in terms of a. [3]

4 A curve C has parametric equations

$$x = t^2 \ , \ y = \frac{1}{\sqrt{t}} \ , t > 0 \ .$$

(i) The curve $y = \frac{8}{x}$ intersects C at point A. Without using a calculator, find the coordinates of A. [2]

(ii) The tangent at the point $P\left(p^2, \frac{1}{\sqrt{p}}\right)$ on C meets the x-axis at point D and the y-axis at point E. The point F is the midpoint of DE. Find a cartesian equation of the curve traced by F as p varies. [5]

5 The equation of a curve is $2xy + (1+y)^2 = x$.

(i) Find the equations of the two tangents which are parallel to the y-axis. [4]

(ii) The normal to the curve at the point $A(1,\ 0)$ meets the y-axis at the point B. Find the area of the triangle OAB. [3]

6 The sum of the first n terms of a sequence is a cubic polynomial, denoted by S_n. The first term and the second term of the sequence are 2 and 4 respectively. It is known that $S_5 = 90$ and $S_{10} = 830$.

(i) Find S_n in terms of n. [4]

(ii) Find the 54th term of the sequence. [2]

(b) **(i)** Given that $\cos(2n-1)\alpha - \cos(2n+1)\alpha = 2\sin\alpha\sin 2n\alpha$ and α is not an integer multiple of π, show that

$$\sum_{n=1}^{N} \sin 2n\alpha = \frac{1}{2}\cot\alpha - \frac{1}{2}\operatorname{cosec}\alpha\cos(2N+1)\alpha .$$ [3]

(ii) Explain whether the series $\sum_{n=1}^{\infty} \sin\frac{2n\pi}{3}$ converges. [1]

7 **(a)(i)** Find $\int \cos(\ln x)\,\mathrm{d}x$. [3]

(ii) A curve C is defined by the equation $y=\cos(\ln x)$, for $\mathrm{e}^{-\frac{3}{2}\pi} \le x \le \mathrm{e}^{\frac{1}{2}\pi}$. The region R is bounded by C, the lines $x=\mathrm{e}^{-\frac{\pi}{2}}$, $x=\mathrm{e}^{\frac{\pi}{2}}$ and the x-axis. Find the exact area of R. [3]

(b) A curve is defined by the equation $y = \frac{\sqrt{e^{\cot x}}}{\sin x}$. The region bounded by this curve, the x-axis, the lines $x = \frac{\pi}{6}$ and $x = \frac{2\pi}{3}$, is rotated 2π radians about the x-axis to form a solid. Using the substitution $u = \cot x$, find the exact volume of the solid obtained. [4]

8 **(i)** Show that $y=\dfrac{x-x^2-1}{x-2}$ can be expressed as $y=\dfrac{A}{x-2}+B(x+1)$, where A and B are constants to be found. Hence, state a sequence of transformations that will transform the curve with equation $y=\dfrac{1}{3-x}-\dfrac{x}{3}$ onto the curve with equation $y=\dfrac{x-x^2-1}{x-2}$. [3]

(ii) On the same axes, sketch the curves with equations $y=\dfrac{x-x^2-1}{x-2}$ and $y=|2x+1|$, stating the equations of any asymptotes and the coordinates of the points where the curves cross the axes. [4]

Hence, find the exact range of values of x for which $\dfrac{x-x^2-1}{x-2}<|2x+1|$. [4]

9 **(a)** The function f is defined by $\mathrm{f}: x \mapsto 2+\dfrac{3}{x},\ x \in \mathbb{R},\ x>0$.

(i) Sketch the graph of $y=\mathrm{f}(x)$. Hence, show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) On the same diagram as in part **(i),** sketch the graphs of $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$. [2]

(iv) Explain why f^2 exists and find $\mathrm{f}^2(x)$. [2]

(b) The function h is defined by $\mathrm{h}: x \mapsto \dfrac{3-x}{x^2-1}$, $x \in \mathbb{R}$, $x \neq \pm 1$. Find algebraically the range of h, giving your answer in exact form. [4]

10 A tank initially contains 2000 litres of water and 20 kg of dissolved salt. Brine with C kg of salt per 1000 litres is entering the tank at 5 litres per minute and the solution drains out at the same rate of 5 litres per minute. The amount of salt in the tank at time t minutes is x kg. Assume that the solution is always uniformly mixed.

(i) Show that $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{400}(2C - x)$. Hence determine the value of C if the amount of salt in the tank remains constant at 120 kg after certain time has passed. [3]

(ii) Find x in terms of t. [5]

(iii) Sketch the graph of the particular solution, including the coordinates of the point(s) where the graph crosses the axes and the equations of any asymptotes. Find the time t when the amount of salt in the tank is 60 kg, giving your answer to the nearest minute. [3]

(iv) State one assumption for the above model to be valid. [1]

11 A factory produces power banks. The factory produces 1000 power banks in the first week. In each subsequent week, the number of power banks produced is 250 more than the previous week. The factory produces 7500 power banks in the Nth week.

(i) Find the value of N. [2]

(ii) After the Nth week, the factory produces 7500 power banks weekly. Find the total number of power banks that will be produced in the first 60 weeks. [3]

The sales manager predicts that the demand for power banks in this week is $a+bH$ if the demand for power banks in the preceding week is H, where a and b are constants and $b>1$. It is given that the demand for power banks in the first week is 50.

(iii) Show that the demand for power banks in the third week is $a+ba+50b^2$. [2]

(iv) Show that the demand for power banks in the nth week can be written as $a\left(\dfrac{b^{n-1}-1}{b-1}\right)+50b^{n-1}$. [1]

It is now given that $a = 300$ and $b = 1.05$.
In the first week, the number of power banks produced is still 1000. In each subsequent week, the number of power banks produced will be L more than the previous week.

(v) The production manager decides to change the production plan so that the total production can meet the total demand in the first 60 weeks. Find the least value of L. [4]

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