Name:		Centre/Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) — 9758/02

Paper 2

19 September 2022

3 hours

Candidates answer on the Question Paper

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**5**	**6**	**7**	**9**	**13**	**7**	**9**	**10**	**9**	**12**	**13**	**100**

Section A: Pure Mathematics [40 marks]

1 **(a)** Sketch the curve C given by the equation

$$4x^2 - 9(y+1)^2 - 36 = 0,$$

indicating clearly the equations of any asymptotes. [2]

(b) Find the volume generated when the region bounded by C and the line $x = -\frac{\sqrt{45}}{2}$ is rotated through 2π radians about the y-axis. [3]

2 **Do not use a calculator in answering this question.**

The complex number w is given by $2-3\text{i}$.

(a) Find w^3 in the form $x+\text{i}y$, showing your working. [2]

w is a root of the equation $z^3 - 5z^2 + az + b = 0$ where a and b are real constants. This equation also has a real root α.

(b) Find the values of a and b. [2]

(c) Find a cubic polynomial that has roots $\text{i}w, \text{i}w^*$ and $\text{i}\alpha$ in the form $z^3 + pz^2 + qz + r = 0$, where p, q and r are real constants. [2]

3 The curve C has equation $y = \dfrac{4(x-2)^2}{x-4}$.

(a) Sketch C. Label the coordinates of any stationary points and point(s) of intersection with the axes. State the equations of its asymptotes and the coordinates of the point of intersection. [3]

(b) Deduce the range of values of a such that the equation

$$(x-4)^2 + \left(\frac{4(x-2)^2}{x-4} - 16\right)^2 = a$$

has a negative real root. [2]

(c) Suppose the variable point P on C represents the complex number $z = x + \text{i}\,y$. By referring to the point of intersection between the asymptotes of C, find the range of values of $\arg(z - 4 - 16\text{i})$. [2]

4 Relative to the origin O, the position vectors of the points P, Q and R are $\mathbf{i}$, $2\mathbf{j} - t\mathbf{k}$ and $t\mathbf{k}$ respectively, where t is a fixed constant. The points A and B divides both line segments PQ and QR respectively in the same ratio of $\mu : 1 - \mu$, where μ is a parameter such that $0 < \mu < 1$.

(a) Find the vector $\overrightarrow{AB}$ in terms of t and μ. [3]

(b) Determine whether the points O, A, B are collinear. [1]

(c) Find the values of μ such that the length of projection of $\overrightarrow{AB}$ onto $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ is $\frac{1}{5}$ unit. [2]

(d) Given that angle AOB is a right angle, find the set of possible values of t, justifying your answer clearly. [3]

5 **(a)** Show, by integration, that $\int \sin x(1-\sin x)\,\mathrm{d}x = -\cos x - \frac{1}{2}x + \frac{1}{4}\sin 2x + D$ where D is an arbitrary constant. [2]

With reference from the origin O, the curve C has parametric equations

$$x = \theta(1-\sin\theta),\ y = 1-\cos\theta, \quad \text{for } 0 \le \theta \le \tfrac{\pi}{2}.$$

The line L has the equation $y = 2x$.

(b) Sketch C, showing clearly the coordinates of any point(s) of intersection with the axes. [2]

(c) A point P lies on C where the tangent at P is parallel to L. Find the coordinates of P. [3]

(d) Find the exact area of the region enclosed by C and the y-axis. [4]

(e) A point Q lies on C such that OQ makes an angle of $\frac{\pi}{6}$ radians with L. Find the value of the parameter θ for Q. [2]

Section B: Probability and Statistics [60 marks]

6 **(a)** Find the number of ways the letters of the word ABDUCTIONS can be arranged in a circle such that the letters A, O and U must not be next to one another. [2]

(b) A five-letter codeword is randomly formed from the word TYRANNOSAUR.

(i) Find the number of codewords that can be formed if it contains at most 1 pair of identical letters. [3]

(ii) Find the probability that a codeword contains both R's and both N's, given that it contains the block of letters "RAN". [2]

7 Commodity X is **traded four times a week from Monday to Thursday**. The unit price of X can only rise or fall on any day. If the unit price of X rises on a day, there is a probability of 0.6 that it will rise on the next trading day. If the unit price of X falls on a day, there is a probability of 0.15 that it will rise on the next trading day.

In a particular week, the unit price of X rises on Monday and the events A and B are defined as follows.

A: the unit price of X falls on Tuesday.
B: the unit price of X rises on Thursday.

(a) Find

(i) $\mathrm{P}(A \cap B)$, [2]

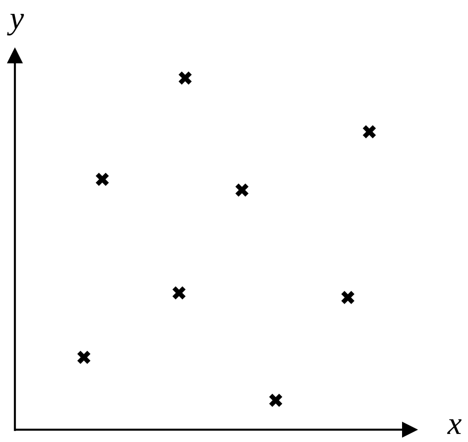
(ii) $\mathrm{P}(B)$, [2]

(iii) $\mathrm{P}(B \mid A)$. [2]

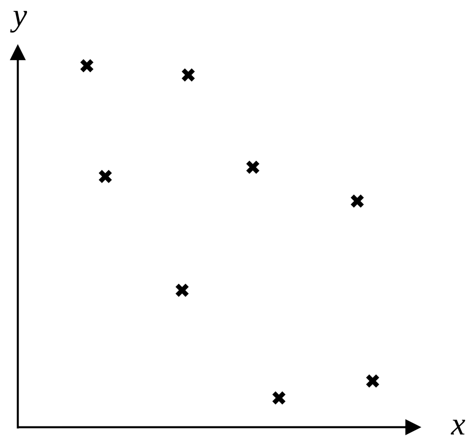
(b) State, with a reason, whether A and B are independent. [1]

(c) If the unit price of X rises for 12 consecutive Mondays in a particular year, find the probability that it rises on exactly 5 of the corresponding Tuesdays. [2]

8 **(a)** The scatter diagrams of two different sets of data points are shown as follows:



Set A



Set B

Determine, with a reason, which data set would result in a larger absolute value of the product moment correlation coefficient. [2]

(b) A researcher investigates the relationship between the yearly mean household income, x thousands, and the yearly mean household expenditure, y thousands. The table below shows the detailed data over 10 years.

x	52.0	54.1	48.8	49.1	45.5	47.9	51.0	50.4	53.5	55.5
y	47.0	53.1	46.0	45.6	45.5	45.0	46.3	46.1	48.0	54.0

(i) Give a sketch of the scatter diagram of the data. [1]

(ii) The researcher proposes two models to represent the data:

$$C: \; y = a + b\ln x$$

$$D: \; y = a + be^{\frac{1}{2}x}$$

By calculating the product moment correlation coefficients for this data, explain which is the more appropriate model. [2]

Use the more appropriate model in part **(ii)** for the rest of the question.

(iii) Use a regression line to obtain the estimate for the household expenditure for a household income of \$50 000, correct to the nearest integer. Comment on the reliability of this estimate. [3]

(iv) Explain if it is valid to conclude that a higher income will result in a higher expenditure. [1]

(v) A student claims that the product moment correlation coefficient will remain the same between the models $y = a + be^{\frac{1}{2}x}$ and $y = p + qe^{\frac{1}{20}x}$. Comment whether the claim is true. [1]

9 **(a)** A confectionery is selling mooncakes during the Mid-Autumn Festival. The owner claims that the mean mass of the mooncakes is at least 150 g. The distribution of the mass of a mooncake has standard deviation 6.73 g. A customer bought a random selection of 9 mooncakes from the confectionery with masses, in g, as follows

145	148	153	156	141	151	143	157	138

Test at the 10% significance level whether the owner's claim is valid. State an assumption about the population distribution of the mass of the mooncakes. [5]

(b) In a particular country, the mean working hours of teachers in schools is 60 hours per week. After implementation of the Home-Based Learning in all schools, the Ministry claims that the mean working hours of teachers remained unchanged. A random sample of 50 teachers was taken and the number of working hours per week was recorded. The mean working hours and standard deviation for the sample were found to be 62 hours and k hours respectively. A hypothesis test was conducted and it was found that there is sufficient evidence to reject the Ministry's claim at 5% significance level. Find the set of values of k. [4]

10 In this question you should state the parameters of any distributions that you use.

A fruit stall vendor sells rock melons and watermelons. The masses (in grams) of a randomly chosen rock melon and watermelon, denoted by X and Y respectively, have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean (g)	Standard deviation (g)
X	580	22
Y	870	30

(a) Sketch the distribution of X for masses between 558 g to 646 g. [1]

(b) Find the expected number of rock melons with mass more than 600 g from 300 randomly chosen rock melons. [2]

(c) Comment whether the combined masses of the melons of both types is normally distributed. [1]

Rock melons and watermelons are priced at \$3/kg and \$2.80/kg respectively.

(d) Find the probability that the mean selling price of 4 randomly chosen rock melons differs from the selling price of a randomly chosen watermelon by at most 60 cents. [4]

(e) Ah Guan buys 20 melons, n of them are rock melons and the rest are watermelons. Find the greatest value of n such that the probability that the total cost of these 20 melons exceeding \$38 is more than 0.95. You should show your working clearly. [4]

11 Emma has a computer program that generates a random positive integer X. The probability distribution of X is :

$$\mathrm{P}(X=r)=\frac{a}{r^3},\quad r\in\mathbb{Z}^+ \text{ and } a \text{ is positive constant.}$$

For the rest of the question, you may use the following results:

$$\sum_{m=1}^{\infty}\frac{1}{m} \text{ does not exist, } \sum_{m=1}^{\infty}\frac{1}{m^2}=1.6449 \text{ and } \sum_{m=1}^{\infty}\frac{1}{m^3}=1.2021.$$

(a) Find the value of a. [2]

(b) Find E(X) and explain why Var(X) cannot be calculated. [2]

(c) Find $\mathrm{P}\left(X\geq 2 \mid X\leq 15\right)$. [3]

Emma generates 10 numbers using her program namely, $X_1, X_2, \ldots, X_{10}$. The random variable Y denotes the number of times the number '3' occurs among the 10 numbers. You can assume that the numbers generated by the program are independent of each other.

(d) Find $\mathrm{P}(Y>2)$. [2]

(e) Find $\mathrm{P}(X_1+Y=3)$. [4]