JURONG PIONEER JUNIOR COLLEGE
JC2 Preliminary Examination 2019

MATHEMATICS 9758/02

Section A : Pure Mathematics [40 Marks]

1 A sequence a_0, a_1, a_2, … is given by $a_0 = \dfrac{3}{5}$ and $a_{n+1} = a_n + 3^n - n$ for $n \geq 0$. By considering $\displaystyle\sum_{r=0}^{n-1}\left(a_{r+1} - a_r\right)$, find a formula for a_n in terms of n. [5]

2 **In this question, you may use expansions from the List of Formula (MF26).**

(a) **(i)** Find the Maclaurin expansion of ln(cos 3x) in ascending powers of x, up to and including the term in x^6. [5]

(ii) Hence, state the Maclaurin expansion of tan 3x, up to and including the term in x^5. [2]

(b) Given that x is sufficiently small, find the series expansion of $\dfrac{e^{\tan x}}{(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [3]

3 A curve C has parametric equations $x = 4\sin 2t$, $y = 4\cos 2t$, where $0 \leq t \leq \dfrac{\pi}{4}$.

(i) Sketch C. [2]

(ii) State the exact value(s) of t at the point(s) where the tangent(s) to C are parallel to the
(a) y-axis,
(b) x-axis. [2]

(iii) P is a point on C where the normal at P is parallel to the line $y = \sqrt{3}x - 2$. Find the equation of the tangent at P, in the form of $y = \dfrac{a}{3}(b - x)$, where a and b are the constants to be determined. [3]

(iv) Find the exact area bounded by C, the tangent at P and the axes. [2]

4 A differential equation is given by $2u^2\dfrac{\mathrm{d}^2x}{\mathrm{d}u^2} + 4u\dfrac{\mathrm{d}x}{\mathrm{d}u} = 15u + 12$ where $x = 0$ and $\dfrac{\mathrm{d}x}{\mathrm{d}u} = 1$ when $u = 1$. By differentiating $u^2\dfrac{\mathrm{d}x}{\mathrm{d}u}$ with respect to u, show that the solution of the differential equation is given by $x = au + \dfrac{b}{u} + c\mathrm{f}(u) + d$, where a, b, c and d are constants to be determined and f(u) is a function in u to be found. [6]

5 The plane p has equation $\mathbf{r}=\begin{pmatrix}1\\0\\-3\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\0\end{pmatrix}+\mu\begin{pmatrix}2\\3\\-4\end{pmatrix}$, and the line l has equation $\mathbf{r}=\begin{pmatrix}-10\\0\\4\end{pmatrix}+t\begin{pmatrix}8\\-4\\1\end{pmatrix}$, where λ, μ and t are parameters.

(i) Show that l is perpendicular to p and find the values of λ, μ and t which give the coordinates of the point at which l and p intersect. [5]

(ii) Find the cartesian equations of the planes such that the perpendicular distance from each plane to p is 2. [5]

Section B : Statistics [60 Marks]

6 On average, 35% of the boxes of Brand *A* cereal contain a voucher. Brandon buys one box of cereal each week. The number of vouchers he obtains is denoted by X.

(i) State, in context, two conditions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X has a binomial distribution. In order to claim a free gift, 8 vouchers are required. Find the probability that Brandon

(ii) obtains at most 3 vouchers in 9 weeks, [1]

(iii) will be able to claim a free gift only in the 10^{th} week. [2]

$100p$ % of the boxes of Brand *B* cereal contain a voucher. Brandon also buys a box of Brand *B* cereal each week for 10 weeks. Given that the probability that Brandon obtains at most 1 voucher in ten weeks is 0.4845, write down an equation in terms of p and hence, find the value of p. [2]

7 A company sells peanut butter in jars. Each jar is labelled as containing m grams of peanut butter on average. A consumer group suspects that the average mass of peanut butter in a jar is overstated. To test this suspicion, the consumer group checks a random sample of 50 jars and the mass of peanut butter per jar, x grams, are summarized by

$$\sum(x-390)=120 \qquad \text{and} \qquad \sum(x-390)^2=3100.$$

The consumer group uses the above data to carry out a test at the 2% level of significance. The result leads the consumer group to conclude that the company has overstated the average mass of peanut butter in a jar.

(i) Explain why the consumer group is able to carry out a hypothesis test without knowing anything about the distribution of the mass of the peanut butter of the jars. [1]

(ii) Find the least possible value of m, to the nearest gram that leads to the result of the hypothesis test as stated above. [7]

8 **(a)** Find the number of different 6-digit numbers that can be formed from the digits 0, 2, 4, 5, 6 and 8, if no digit is repeated and the numbers formed are divisible by 5. [2]

(b) Find how many 4-letter code words can be formed from the letters of the word ***DIFFERENT***. [4]

9 In this question you should state the parameters of any distributions that you use.

The masses of Grade *A* and Grade *B* strawberries are normally distributed with mean 18 grams and 12 grams respectively and standard deviation 3 grams and 2 grams respectively.

(i) Find the probability that a randomly chosen Grade *A* strawberry has mass between 17 and 20 grams. [1]

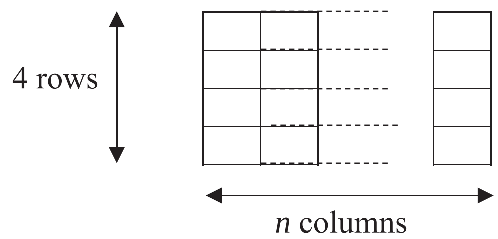
(ii) Any Grade *B* strawberry that weighs less than m grams will be downgraded to Grade *C*. Given that there is a probability of at least 0.955 that a Grade *B* strawberry will not be downgraded to Grade *C*, find the greatest value of m. [2]

Grade *A* strawberries are packed into bags of 12 while Grade *B* strawberries are packed into bags of 15.

(iii) Find the probability that a bag of Grade *A* strawberries weighs more than a bag of Grade *B* strawberries. [3]

(iv) State an assumption needed for your calculation in part **(iii)**. [1]

10 At a particular booth in a funfair, Kathryn is given some boxes which are arranged in the layout as shown below.



Each box contains a numbered card. One card is numbered '4', three cards are numbered '2', and the rest of the cards are numbered '1'. All the boxes are closed initially and Kathryn is required to open 2 different boxes. Her score is the sum of the numbers obtained.

If she scores more than 4, she wins \$10. If she scores less than 4, she loses \$2. If she scores 4, she does not win anything. The random variable W is her winnings after one game.

(i) Show that $\mathrm{P}(W = 10) = \dfrac{1}{2n}$. [2]

(ii) Given that $\mathrm{P}(W = 10) = \dfrac{1}{8}$, find the value of n. Hence, find the probability distribution of W. [4]

(iii) Find $\mathrm{E}(W)$ and $\mathrm{Var}(W)$. Explain whether Kathryn should play the game. [4]

50 other participants play the game. Find the probability that the mean winnings is at most \$1. [2]

11 **[Leave your answers in fraction]**

The events A and B are such that $\mathrm{P}(A|B) = \dfrac{7}{10}$, $\mathrm{P}(B|A) = \dfrac{4}{15}$ and $\mathrm{P}(A \cup B) = \dfrac{3}{5}$.

Find the exact values of

(i) $\mathrm{P}(A \cap B)$, [3]

(ii) $\mathrm{P}(A' \cap B)$. [2]

For a third event C, it is given that $\mathrm{P}(C) = \dfrac{3}{10}$ and that A and C are independent.

(iii) Find $\mathrm{P}(A' \cap C)$. [2]

(iv) Hence state an inequality satisfied by $\mathrm{P}(A' \cap B \cap C)$. [1]

12 As part of a medical research on diabetes, a team of researchers conducts a study to investigate the amount of glucose y, measured to the nearest 0.5 mg/dl, present in human bodies at different age x, measured in years. The results are given in the table.

x	20	28	36	44	52	60	68	76
y	86.0	90.5	94.5	97.5	100.0	105.0	103.5	104.0

(i) Calculate the product moment correlation coefficient between x and y and explain whether your answer suggests that a linear model is appropriate. [2]

(ii) Draw a scatter diagram for the data, labelling the axes clearly. [1

One of the values of y appears to be incorrect.

(iii) Circle the corresponding point on your diagram and label it P. [1]

For part **(iv)** and **(v)** of this question, you should omit P.

(iv) Explain from your scatter diagram why the relationship between x and y should not be modelled by an equation of the form $y = ax + b$. [1]

(v) Suppose that the relationship between x and y is modelled by an equation of the form $y = c + d\ln x$, where c and d are constants. Find the product moment correlation coefficient between y and $\ln x$ and the constants c and d. [2]

Assume that the value of x at P is correct.

(vi) Use the model $y = c + d\ln x$, with the values of c and d found in **(v)** to estimate the correct value of y at P, giving your answer to the nearest 0.5 mg/dl. Explain why you would expect this estimate to be reliable. [2]

(vii) If the correct value of y at P is used, the 8 data points may be fitted by the model $y = 43.942 + 14.079\ln x$. Find the correct value of y at P, giving your answer to the nearest 0.5 mg/dl. [3]