# TAMPINES MERIDIAN JUNIOR COLLEGE

## JC2 PRELIMINARY EXAMINATION

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| CANDIDATE NAME | |
| CIVICS GROUP | |

**H2 MATHEMATICS** **9758/01**
Paper 1 **18 September 2019**
**3 hours**

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

| For Examiners' Use | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| Total | |

**[Turn Over**

**1** Using an algebraic method, solve the inequality $\dfrac{4x^2-x-1}{(2x-1)(x+1)} \geq 1$. [4]

Hence or otherwise, solve the inequality

$$\frac{4e^{2x}-e^x-1}{(2e^x-1)(e^x+1)} \geq 1,$$

leaving your answers in exact form. [2]

**2** Referred to the origin $O$, $A$ is a fixed point with position vector $\mathbf{a}$, and $\mathbf{d}$ is a non-zero vector. Given that a general point $R$ has position vector $\mathbf{r}$ such that $\mathbf{r}\times\mathbf{d}=\mathbf{a}\times\mathbf{d}$, show that $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$, where $\lambda$ is a real constant. Hence give a geometrical interpretation of $\mathbf{r}$. [3]

Let $\mathbf{a}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}2\\1\\5\end{pmatrix}$. By writing $\mathbf{r}$ as $\begin{pmatrix}x\\y\\z\end{pmatrix}$, use $\mathbf{r}\times\mathbf{d}=\mathbf{a}\times\mathbf{d}$ to form three equations which represent cartesian equations of three planes. State the relationship between these three planes. [3]

**3** **(i)** The sum of the first $n$ terms of a sequence is denoted by $S_n$. It is known that $S_5=-30$, $S_{14}=168$ and $S_{18}=9S_{10}$. Given that $S_n$ is a quadratic polynomial in $n$, find $S_n$ in terms of $n$. [4]

**(ii)** The $n$th term of the sequence is denoted by $T_n$. Find an expression for $T_n$ in terms of $n$. Hence find the set of values of $n$ for which $|T_n|<12$. [4]

**4** **(i)** Given that f is a strictly increasing continuous function, explain, with the aid of a sketch, why

$$\frac{1}{n}\left\{f\left(\frac{0}{n}\right)+f\left(\frac{1}{n}\right)+\ldots+f\left(\frac{n-1}{n}\right)\right\}<\int_0^1 f(x)\ \mathrm{d}x,$$

where $n$ is a positive integer. [3]

**(ii)** Hence find the least exact value of $k$ such that $\dfrac{1}{n}\left(e^{\frac{0}{n}}+e^{\frac{2}{n}}+e^{\frac{4}{n}}+\ldots+e^{\frac{2n-2}{n}}\right)<k$, where $n$ is a positive integer. [2]

**5** It is given that $\mathrm{f}(x)=4x-x^3$.

**(i)** On separate diagrams, sketch the graphs of $y=|\mathrm{f}(x)|$ and $y=\mathrm{f}(|x|)$, showing clearly the coordinates of any axial intercept(s) and turning point(s). [4]

**(ii)** Find the exact value of the constant $k$ for which $\int_0^k |\mathrm{f}(x)|\,\mathrm{d}x=\int_{-2}^2 \mathrm{f}(|x|)\,\mathrm{d}x$. [4]

**6** Show that $2\cos(r\theta)\sin\theta \equiv \sin[(r+1)\theta]-\sin[(r-1)\theta]$. [1]

By considering the method of differences, find $\sum_{r=1}^{n}\cos(r\theta)$ where $0<\theta<\dfrac{\pi}{2}$.

(You need not simplify your answer.) [3]

Hence evaluate the sum

$$\cos\left(\frac{19}{6}\pi\right)+\cos\left(\frac{20}{6}\pi\right)+\cos\left(\frac{21}{6}\pi\right)+\cdots+\cos\left(\frac{56}{6}\pi\right)+\cos\left(\frac{57}{6}\pi\right),$$

leaving your answer in exact form. [4]

**7** The function f is defined by

$$\mathrm{f}(x)=\begin{cases} n-x & \text{for } n\le x<n+1, \text{where } n \text{ is any positive odd integer,}\\ x-\dfrac{n}{2} & \text{for } n\le x<n+1, \text{where } n \text{ is any positive even integer.}\end{cases}$$

**(i)** Show that $\mathrm{f}(1.5)=-0.5$ and find $\mathrm{f}(2.5)$. [2]

**(ii)** Sketch the graph of $y=\mathrm{f}(x)$ for $1\le x<5$. [2]

**(iii)** Does f have an inverse for $1\le x<5$? Justify your answer. [2]

**(iv)** The function g is defined by $\mathrm{g}:x\mapsto\dfrac{2x-1}{x+1}$, $x\in\mathbb{R}$, $x\ne-1$. For $2\le x<3$, find an expression for $\mathrm{gf}(x)$ and hence, or otherwise, find $(\mathrm{gf})^{-1}\left(\dfrac{2}{3}\right)$. [4]

**[Turn Over**

**8** At the start of an experiment, a particular solid substance is placed in a container filled with water. The solid substance will begin to gradually dissolve in the water. Based on experimental data, a student researcher guesses that the mass, $x$ grams, of the remaining solid substance at time $t$ seconds after the start of the experiment satisfies the following differential equation

$$\frac{dx}{dt} = \frac{1}{k-1}(x-1)(x-k),$$

where $k$ is a real constant and $k > 3$.

**(i)** Show that a general solution of this differential equation is $\ln\left|\frac{x-k}{x-1}\right| = t + C$, where $C$ is an arbitrary real constant. [3]

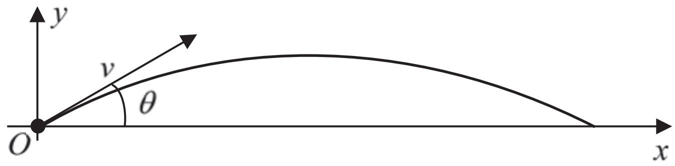
For the rest of the question, let $k = 4$. It is given that the initial mass of the solid substance is 3 grams.

**(ii)** Express the particular solution of the differential equation in the form $x = f(t)$. [4]

**(iii)** Find the exact time taken for the mass of the solid substance to become half of its initial value. [2]

**(iv)** Sketch the part of the curve with the equation found in part **(ii)** which is relevant in this context. [2]

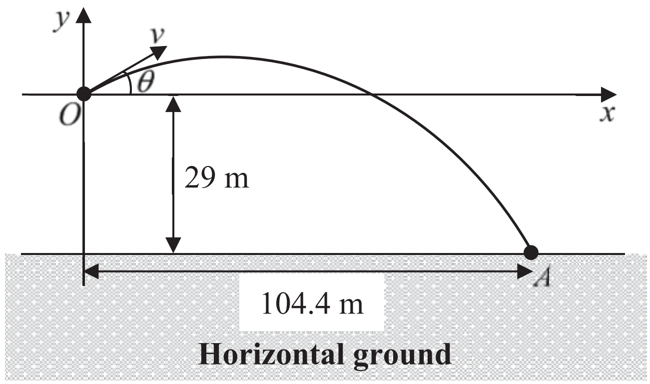
**9**




From a point $O$, a particle is projected with velocity $v\ \text{ms}^{-1}$ at a fixed angle of elevation $\theta$ from the horizontal, where $v$ is a positive real constant and $0 < \theta < \frac{\pi}{2}$. The horizontal displacement, $x$ metres, and the vertical displacement, $y$ metres, of the particle at time $t$ seconds may be modelled by the parametric equations

$$x = (v\cos\theta)t, \qquad y = (v\sin\theta)t - 5t^2.$$

**(i)** Using differentiation, find the maximum height achieved by the particle in terms of $v$ and $\theta$. (You need not show that the height is a maximum.) [3]




The particle is now projected from point $O$ situated at a height of 29 m above the horizontal ground. The particle hits the ground at $A$ which is at a horizontal distance of 104.4 m from $O$. The maximum height (measured from horizontal ground) that the particle reaches is 57.8 m. The diagram above shows the path of the particle (not drawn to scale).

**(ii)** Find the time taken for the particle to hit the ground at $A$ and find the corresponding value of $v$. [5]

**(iii)** Find the exact gradient of the tangent at $A$. [2]

**[Turn Over**

**10** Two houses, $A$ and $B$, have timber cladding on the end of their shed roofs, consisting of rectangular planks of decreasing length.

**(i)** The first plank of the roof of house $A$ has length 350 cm and the lengths of the planks form a geometric progression. The $20^{th}$ plank has length 65 cm. Show that the total length of all the planks must be less than 4128 cm, no matter how many planks there are. [4]

House $B$ consists of only 20 planks which are identical to the first 20 planks of house $A$.

**(ii)** The total length of all the planks used for house $B$ is $L$ cm. Find the value of $L$, leaving your answer to the nearest cm. [2]

**(iii)** Unfortunately the construction company misunderstands the instructions and covers the roof of house $B$ wrongly, so that the lengths of the planks are in arithmetic progression with common difference $d$ cm. If the total length of the 20 planks is still $L$ cm and the length of the $20^{th}$ plank is still 65 cm, find the value of $d$ and the length of the longest plank. [4]

It is known that house $C$ has timber cladding on the end of its shed roof, consisting of rectangular planks of increasing length. The first plank of the roof of house $C$ has length 65 cm and the lengths of the planks are in arithmetic progression with common difference 11 cm. The total length of the first $N$ planks of the roof of house $C$ exceeds 20 640 cm. Find the least value of $N$. [3]

**11** A circle with a fixed radius $r$ cm is inscribed in an isosceles triangle $ABC$ where $\angle ABC = \theta$ radians and $AB = BC$. The circle is in contact with all three sides of the triangle at the points $D$, $E$ and $F$, as shown in Fig. 1.

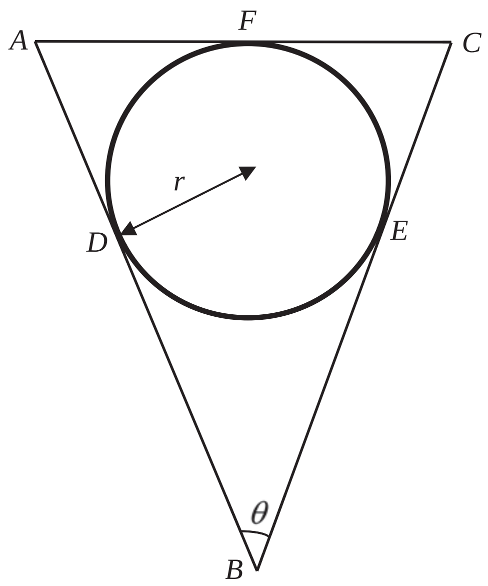



**Fig. 1**

**(i)** Show that the length $BD$ can be expressed as $r\cot\dfrac{\theta}{2}$ cm. [1]

**(ii)** By finding the length $AD$ in terms of $r$ and $\theta$, show that the perimeter of the triangle $ABC$ can be expressed as $4r\cot\left(\dfrac{\pi}{4}-\dfrac{\theta}{4}\right)+2r\cot\dfrac{\theta}{2}$ cm. [2]

**(iii)** Using differentiation, find the exact value of $\theta$ such that the perimeter of the triangle $ABC$ is minimum. Find the minimum perimeter of triangle $ABC$, leaving your answer in the form $a\sqrt{b}r$ cm, where $a$ and $b$ are positive integers to be determined. [6]

**[Turn Over**

Fig. 2 shows a decorative item in the shape of a sphere with a fixed radius inscribed in an inverted right circular cone with base radius $R$ cm and slant height $2R$ cm. The sphere is in contact with the slopes and the base of the cone.

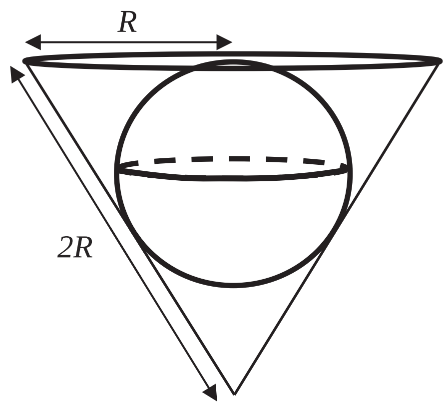



**Fig. 2**

To make the item glow in the dark, the sphere is filled entirely with fluorescent liquid. However, due to a manufacturing defect, the fluorescent liquid leaks into the bottom of the inverted cone at a rate of 2 $cm^3$ per minute.

**(iv)** Assuming that the leaked liquid in the inverted cone will not reach the exterior of the sphere, find the exact rate of increase of the depth of the leaked liquid in the inverted cone when the volume of the leaked liquid in the inverted cone is $24\pi$ $cm^3$. Express your answer in terms of $\pi$. [6]

[The volume of a cone of base radius $r$ and height $h$ is given by $\frac{1}{3}\pi r^2 h$.]

**End of Paper**