YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME

CG | INDEX NO

MATHEMATICS

9758/02

Paper 2

14 September 2020

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6
Marks						

Question	7	8	9	10	Total marks	100
Marks						

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Section A: Pure Mathematics [40 marks]

1 **(i)** State the derivative of $\tan\left(x^3\right)$. [1]

(ii) Find $\int 6x^5 \sec^2\left(x^3\right) \mathrm{d}x$. [3]

2 **(a)** Given that $z=\dfrac{\lambda-4\mathrm{i}}{1-\lambda\mathrm{i}}$, $\lambda\in\mathbb{R}$ and $\arg(z)=\pi$, find the value of z. [3]

(b) **Do not use a calculator in answering this question.**

The complex numbers z and w are given by

$$z=\frac{1+\mathrm{i}}{1-\mathrm{i}} \text{ and } w=\frac{\sqrt{2}}{1+\mathrm{i}}.$$

(i) Express z and w in exact polar form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r>0$ and $-\pi<\theta\le\pi$. [2]

(ii) On an Argand diagram, mark the points A, B and C representing z, w and $z+w$ respectively. State the shape of the quadrilateral $OACB$. [3]

(iii) Hence by finding the argument of $z+w$, show that

$$\tan\frac{\pi}{8}=\sqrt{2}-1.$$ [3]

3 A curve C has parametric equations

$$x = 3\cos\theta - 2\cos 3\theta\ ,$$
$$y = 9\sin\theta - \sin 3\theta\ ,$$

for $0 \le \theta \le 2\pi$.

(i) Sketch C and state the cartesian equations of its lines of symmetry. [2]

(ii) Given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, find the exact values of θ at the points where C meets the y-axis. [2]

(iii) Show that the area enclosed by the axes and the part of C in the first quadrant is given by

$$\int_{\theta_1}^{\theta_2} (27\cos^2\theta - 27\cos\theta\cos 3\theta + 6\cos^2 3\theta)\ \mathrm{d}\theta,$$

where the values of θ_1 and θ_2 should be stated. [3]

(iv) Hence find the exact total area enclosed by C. [5]

4 The plane π_1 has equation $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = 11$. The plane π_2 passes through the points $A(1, -1,\ 2)$, $B(2,\ 3,\ 4)$ and $C(3,\ 4,\ 3)$.

(i) Find a cartesian equation of π_2. [3]

(ii) Find a vector equation of the line l where π_1 and π_2 meet. [2]

(iii) *D* is a general point on *l*. Find an expression for the square of the distance *AD*. Hence, or otherwise, find the coordinates of the point on *l* which is nearest to *A*. [5]

4 **[Continued]**

(iv) The plane π_3 is parallel to π_1 and contains the point $E(1, k, 3)$. Given that the distance between π_1 and π_3 is $\frac{16}{\sqrt{30}}$, find the possible values of *k*. [3]

Section B: Probability and Statistics [60 marks]

5 An interest group of 8 members consists of one married couple together with 4 other men and 2 other women. Find the number of ways that the 8 people can sit at a round table if

(i) the married couple is to be seated together, [2]

(ii) all women are to be separated and the seats are numbered. [2]

An executive committee consisting of a Chairman, Vice-Chairman and Secretary is to be selected from the group of 8 members.

(iii) Find the number of ways that the executive committee can be formed if at most one person from the married couple can be included. [3]

6 For events A and B, it is given that $\mathrm{P}(A \cup B') = \dfrac{2}{3}$ and $\mathrm{P}(A) = \dfrac{1}{4}$.

(i) Find $\mathrm{P}(A \mid A \cup B)$. [2]

(ii) Given that events A and B are independent, find P(B). [2]

(iii) Given instead that events A and B are **not** independent, find the range of possible values of $\mathrm{P}(B)$. [3]

7 A 6-sided die has three faces marked with a '1', two faces marked with a '2' and one face marked with a '3'. Another 6-sided die has one face marked with a '1', two faces marked with a '2' and three faces marked with a '3'.
In a game, a player throws both dice simultaneously.

- If the numbers on the top faces are the same, two cards are drawn at random without replacement from a bag containing 3 blue cards and 2 white cards.
- If the numbers on the top faces are different, cards will be drawn at random from the bag, one at a time without replacement, until 2 white cards are taken out.

The number of blue cards drawn from the bag is denoted by X.

(i) Show that the probability that the numbers on the top faces are the same is $\frac{5}{18}$. [1]

(ii) Determine the probability distribution of X. [4]

(iii) A player plays 2 games and X_1 and X_2 denote the number of blue cards obtained in the first and second game respectively. Find $\mathrm{P}\left(\left|X_1 - X_2\right| \geq 1\right)$. [2]

(iv) Find the probability that the sum of 30 independent observations of X is greater than 45. [3]

8 A manufacturing company produces surgical masks. The surgical masks are randomly packed into boxes of 50. On average, 15% of the surgical masks are defective. The number of surgical masks that are defective in a box of 50 pieces is denoted by X.

(i) State, in context, two assumptions needed for the number of defective surgical masks in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of defective surgical masks in a box has a binomial distribution.

In a randomly chosen box of 50 surgical masks, find the probability that there are

(ii) more than 8 defective surgical masks, [2]

(iii) at most 12 defective surgical masks given that there are more than 8 defective surgical masks. [3]

For shipping purposes, the boxes are packed into cartons, with each carton containing 24 boxes.

(iv) Find the probability that, in a randomly chosen carton, there are at least 10 boxes containing at most 8 defective surgical masks. [2]

The company also manufactures reusable masks which are packed into packets of 10. The probability that a reusable mask is defective is p. It is known that the modal number of defective reusable masks in a packet is 1.

(v) Use this information to find exactly the range of values that p can take. [3]

9 The mass, X grams, of soup packed into a can by machine A is normally distributed with mean μ grams and standard deviation σ grams.

(i) Given that $\mathrm{P}(X < 112) = \mathrm{P}(X > 128) = 0.09121$, find the values of μ and σ. [3]

The settings of machine A are adjusted so that the mass of soup, in grams, packed into a can is now normally distributed with mean 125 grams and standard deviation 8 grams.

(ii) 3 cans of soup are randomly chosen. Find the probability that 2 of the cans each have a mass greater than 124 grams and the other has a mass between 120 grams and 123 grams. [2]

(iii) A random sample of n cans of soup is taken. Find the least value of n such that the probability that the sample mean mass of soup in the cans exceeds 127 grams is at most 0.1. [3]

The mass, Y grams, of soup packed into a packet by machine B is normally distributed with mean 150 grams and standard deviation 7.5 grams.

(iv) Find the probability that the total mass of soup in 4 randomly chosen packets differs from 1.5 times the total mass of soup in 3 randomly chosen cans by at most 50 grams. [4]

10 A company produces reinforcing bars for use in construction. The production manager claims that the mean tensile strength of the reinforcing bars is at least 620 MPa. The company director wishes to investigate this claim and selects a random sample of 80 reinforcing bars. The tensile strengths, x, in MPa, of the random sample are summarised by

$$\sum(x-620)=-300, \qquad \sum(x-620)^2=36680.$$

(i) Find unbiased estimates of the population mean and variance of the tensile strength of the reinforcing bars. [2]

(ii) Carry out the test, at the 5% level of significance, for the company director. You should state your hypotheses and define any symbols you use. [5]

(iii) Explain what you understand by the phrase 'at the 5% level of significance' in the context of this question. [1]

The quality control manager wishes to test whether the mean tensile strength differs from 620 MPa. The quality control manager finds that the mean tensile strength of 10 randomly chosen reinforcing bars is 628.9 MPa. He carries out a hypothesis test at the 5% level of significance.

(iv) State an assumption needed for the test to be valid. [1]

10 [Continued]

(v) Given that the assumption made in part **(iv)** is true and the quality control manager concludes that the mean tensile strength differs from 620 MPa, find the set of possible values of the population variance used in this test. [3]

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