

Answer all the questions.

1 The quadratic equation  $3x^2 - 5x - 1 = 0$  has roots  $\alpha$  and  $\beta$ .

Find

(i) the value of  $\alpha^2 + \beta^2$

[3]

(ii) a quadratic equation whose roots are  $\frac{4}{\alpha^2}$  and  $\frac{4}{\beta^2}$ .

[3]

- 2 (i) Express  $\frac{6 + 5x - 8x^2}{(2x - 1)(x^2 + 3)}$  as partial fractions.

[4]

(ii) Differentiate  $\ln(x^2 + 3)$  with respect to  $x$ .

[1]

(iii) Using the results of parts (i) and (ii), find  $\int \frac{6 + 5x - 8x^2}{(2x - 1)(x^2 + 3)} dx$ .

[4]

3 (a) Given that  $\log_2 p = x$  and  $\log_2 q = y$ , express the following in terms of  $x$  and  $y$ .

(i)  $\log_2 1 - p + q$  [1]

(ii)  $\log_2 \sqrt{\frac{p^5}{q^3}}$  [3]

(iii)  $\log_{\sqrt{2}} 4p$  [4]

(b) Solve  $4\log_4 x + 1 = 3\log_8(5 - 3x)$ .

[5]

- 4 (i) A curve has the equation  $y = 2x^2 + 4x - 6$ . Express  $y = 2x^2 + 4x - 6$  in the form  $y = a(x+h)^2 + k$ , where  $a$ ,  $h$  and  $k$  are constants. [2]
- (ii) State the coordinates of the minimum point of the curve. [1]
- (iii) Explain how you find the minimum point of the curve. [1]
- (iv) Hence, sketch the graph of  $y = |2x^2 + 4x - 6|$ , indicating the minimum point and the intercept(s) [3]

5 A circle,  $C_1$ , has the equation  $x^2 + y^2 - 4x + 6y - 12 = 0$ .

(i) Find the coordinates of the centre and radius of the circle. [3]

(ii) Find the coordinates of the points where the circle cuts the  $x$  axis. [2]

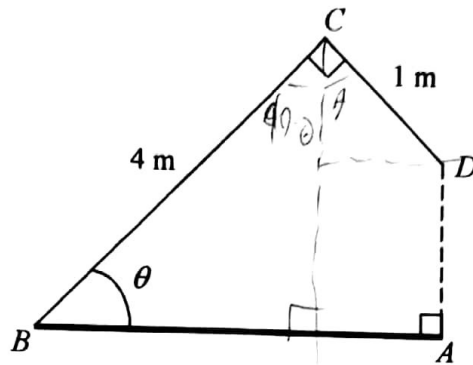
(iii) Find the equation of another circle such that the new circle is a reflection of  $x^2 + y^2 - 4x + 6y - 12 = 0$  along the  $y$ -axis. [1]

(iv) State whether the point  $(4, 1)$  is found inside, on the circumference or outside of circle  $C_1$ . Give reasons for your answer. [2]

6 (i) Show that  $\frac{1}{\sin x + 1} - \frac{1}{\sin x - 1} = 2\sec^2 x$ . [4]

(ii) Hence solve  $\frac{1}{\sin x + 1} - \frac{1}{\sin x - 1} = 3 - \tan x$  where  $0 \leq x \leq 2\pi$ . [5]





The diagram above shows the side view of a bus stop shelter  $BCD$  such that  $BC = 4$  m,  $CD = 1$  m, angle  $BCD = 90^\circ$  and angle  $CBA = \theta$ .  $AB$  is a concrete pavement under the shelter such that  $DA$  is perpendicular to  $AB$ .

(i) Show why  $AB = 4\cos\theta + \sin\theta$ .

[2]

(ii) Express  $AB$  in the form of  $R\cos(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ .

[3]

(iii) State the maximum value of  $AB$  and the value of  $\theta$  at which this occurs.

[3]

(iv) Find the value of  $\theta$  when  $AB = 3$  m.

[2]

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f

H

8 A particle  $P$  moves in a straight line so that,  $t$  seconds after passing through a fixed point  $O$ , its velocity  $v$  m/s, is given by  $v = 3t^2 + kt + 18$ , where  $k$  is a constant. When  $t = 1$ , the acceleration of the particle is  $-9$  m/s<sup>2</sup>.

(i) Show that  $k = -15$ . [2]

(ii) Find the values of  $t$  for which  $P$  is instantaneously at rest. [2]

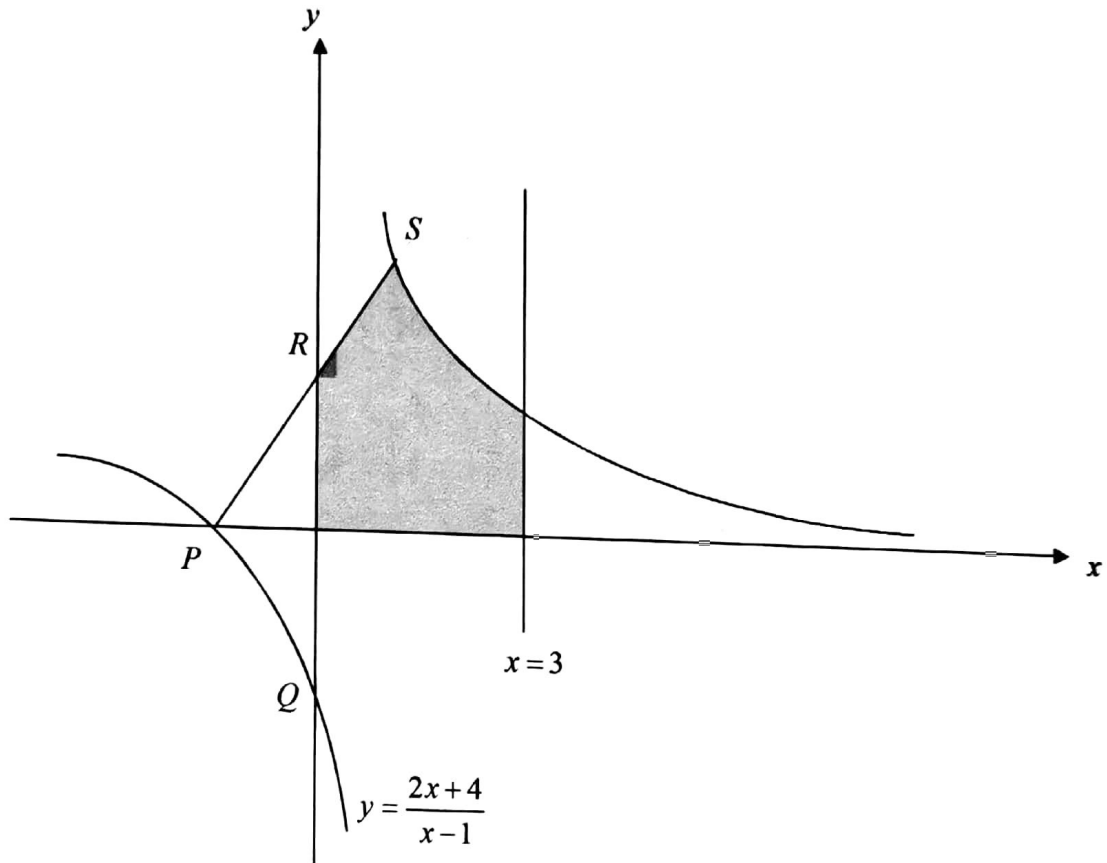
(iii) Find the total distance travelled by  $P$  in the first 3 seconds after passing through point  $O$ . [4]

9 Two variables  $x$  and  $y$  are connected by the equation  $y = \frac{3x+1}{1-2x}$ .

- (i) Find  $\frac{dy}{dx}$ . State the value of  $x$  for which  $\frac{dy}{dx}$  is not defined and explain why  $\frac{dy}{dx}$  is not defined for this value of  $x$ . [4]

- (ii) Is  $y$  an increasing or a decreasing function? Explain your answer. [2]

- 10 The diagram below shows the curve  $y = \frac{2x+4}{x-1}$  which cuts the  $x$ -axis at  $P$  and the  $y$ -axis at  $Q$ . The normal to the curve at  $P$  meets the  $y$ -axis at  $R$ .  $S$  is the point where the normal meets the curve again.



- (i) Find the coordinates of  $P$  and of  $Q$ .

[2]

$$f(x) = \frac{2x+4}{x+1}$$

16

(ii) Find the coordinates of  $R$  and of  $S$ .

[7]

- (iii) Express  $y = \frac{2x+4}{x-1}$  in the form of  $y = A + \frac{B}{x-1}$  and hence find the area of the shaded region. [5]

11 Answer the whole of this question on the next page.

The table below shows experimental values of two variables  $x$  and  $y$ , which are connected by the equation  $y = ax + \frac{b}{x}$  where  $a$  and  $b$  are constants. It is suspected that an error occurred in one of the values of  $y$ .

$x$	0.5	1.0	1.5	2.0	2.5	3.0
$y$	11.2	8.0	8.1	11.0	10.2	11.7

- (i) Plot  $xy$  against  $x^2$  for the given data and draw a straight line graph. [3]
- (ii) Use your graph to estimate the value of  $a$  and of  $b$ . [4]
- (iii) Find the inaccurate reading of  $y$  and estimate its correct value using your graph. [3]