CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination in preparation for
the General Certificate of Education Ordinary Level 2020

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS

4047/01

Paper 1

2 September 2020

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The function $f(x)$ is defined by $f(x)=\dfrac{5x+2}{2x-3}$ for all values of x, $x \neq \dfrac{3}{2}$.

Determine, with working, whether f(x) is an increasing function or a decreasing function. [3]

2 The value of a newly mined diamond increases by half its value every 10 years. It is given that V_o is the value of the diamond at a particular time and V is its value t years later. Find the value of the constant m in the relationship $V=V_o e^{mt}$. [3]

3 It is given that $\lg(a+2)=\log_{100}(b-1)$.

(i) Express b in terms of a. [3]

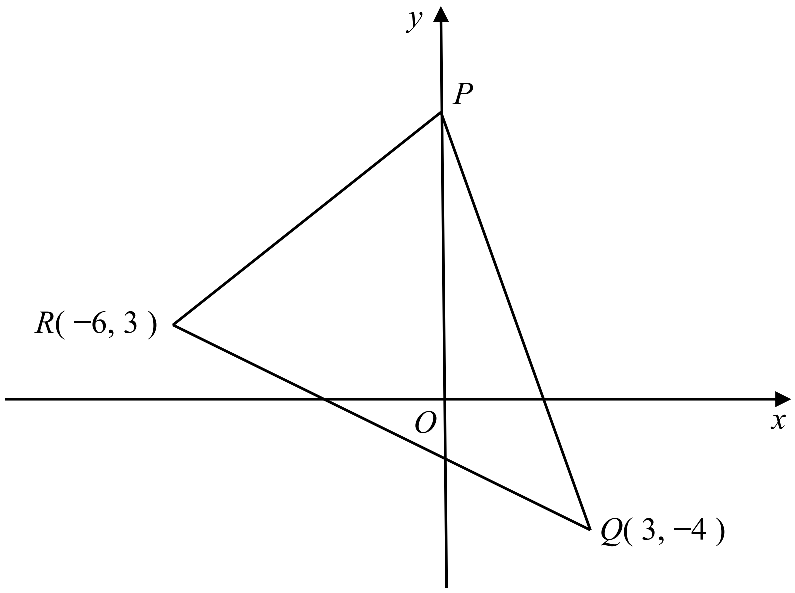
(ii) Given that $b \geq 10$, find the range of values for a. [3]

4 The equation of a curve is $y = kx\sqrt{2x+3}$ where k is a constant.

(i) Obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the form $\frac{ak(x+b)}{\sqrt{2x+3}}$ where a and b are integers. [3]

(ii) A point moves along the curve in such a way that when $x = 3$, the rate of increase of y with respect to time is thrice the rate of increase of x with respect to the time. Find the value of k. [2]

5



In the diagram, PQR is an isosceles triangle in which $PQ = QR$ and the coordinates of Q and R are (3, −4) and (−6, 3) respectively. P is a point on the y-axis.

(i) Find the coordinates of P. [3]

(ii) Find the equation of the line which passes through Q and the midpoint of PR. [3]

6 **(i)** Prove that $\dfrac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}=\dfrac{1}{2}(2-\sin 2\theta)$. [3]

(ii) Hence solve the equation $\dfrac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}=\dfrac{5}{2}-2\sin^2 2\theta$ for $0°\le\theta\le 180°$. [4]

7 **(i)** Sketch the graph of $y = |2x - 5|$. [2]

(ii) Find the x-coordinates of the points of intersection of the graph of $y = |2x - 5|$ and the line $y = 12 - x$. [2]

(iii) Determine the range of values of m such that $|2x - 5| = -|3x| + m$ has one or more solutions. [1]

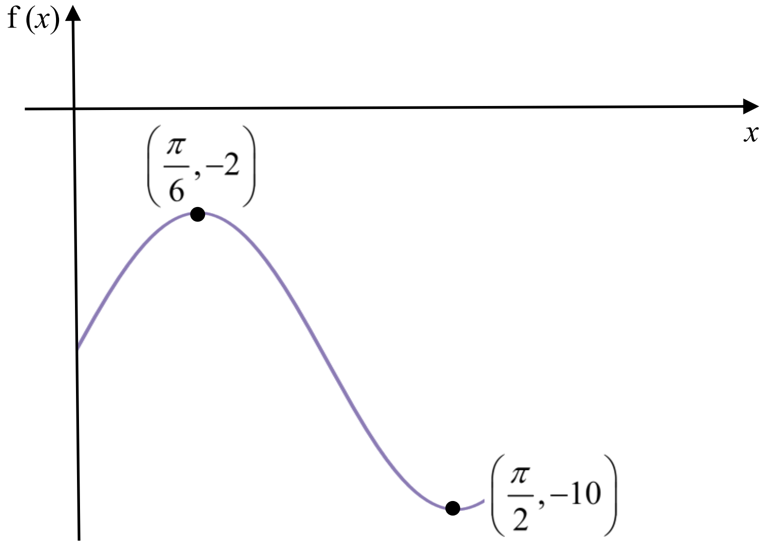
8 **(i)** Find the range of values of p such that $y = px^2 - 4x + p$ lies entirely above the x-axis. [4]

(ii) Explain clearly why the line $y = x + 2k$ will intersect the curve $2y^2 - x^2 = 8$ at two distinct points for all real values of k. [5]

9 **(i)** On the same diagram, sketch the curves $y = \frac{1}{9}x^{\frac{5}{2}}$ and $y = 9x^{\frac{1}{2}}$ for $x \geq 0$. [2]

(ii) Show that the tangent to the curve $y = 9x^{\frac{1}{2}}$ at the point of intersection of the two curves passes through the point (−1, 12). [5]

10 **(a)**



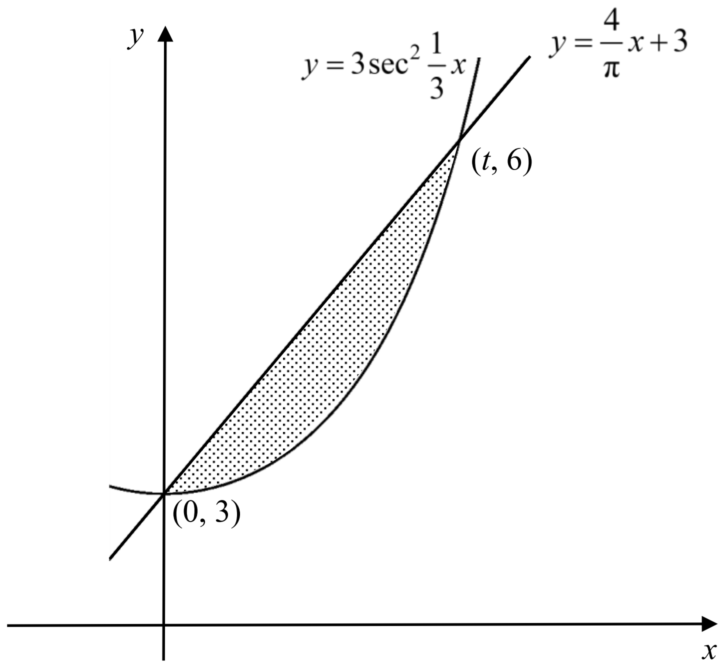
The diagram shows part of the curve $f(x) = 4\sin ax - b$.

The coordinates of the turning points shown are $\left(\frac{\pi}{6}, -2\right)$ and $\left(\frac{\pi}{2}, -10\right)$.

Find the value of a and of b. [2]

(b) Given that $g(x) = 2\sin 3x - 5$, express $g(x)$ as a cubic function in terms of $\sin x$. [5]

11



The diagram shows part of the curve $y = 3\sec^2 \frac{1}{3}x$ and the straight line $y = \frac{4}{\pi}x + 3$.
The line intersects the curve at $(t, 6)$ and $(0, 3)$.

(i) Find the value of t, in terms of π. [1]

(ii) Hence find the area of the shaded region, expressing your answer in terms of π. [4]

12 A particle moves in a straight line so that, t seconds, after leaving a fixed point O, its velocity, v cm/s, is given by $v = 6t^2 - (5k-4)t + 8 - 13h$ where h and k are constants. The acceleration of the particle, 3 seconds after leaving O is 30 cm/s^2.

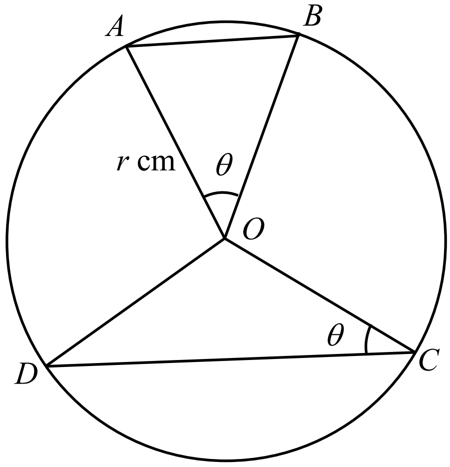
(i) Show that $k = 2$. [2]

The displacement of the particle from O is -500 cm, 2 seconds after leaving O.

(ii) Find the time when the particle returns to O. [5]

(iii) Explain clearly why the particle changes its direction only once. [2]

13



The diagram shows a circular mosaic tile of radius r cm and centre O. The design consists of two triangles OAB and OCD.
Angle AOB = angle OCD = θ radians.

(i) Show that the total area, A cm^2, of triangles OAB and OCD is given by

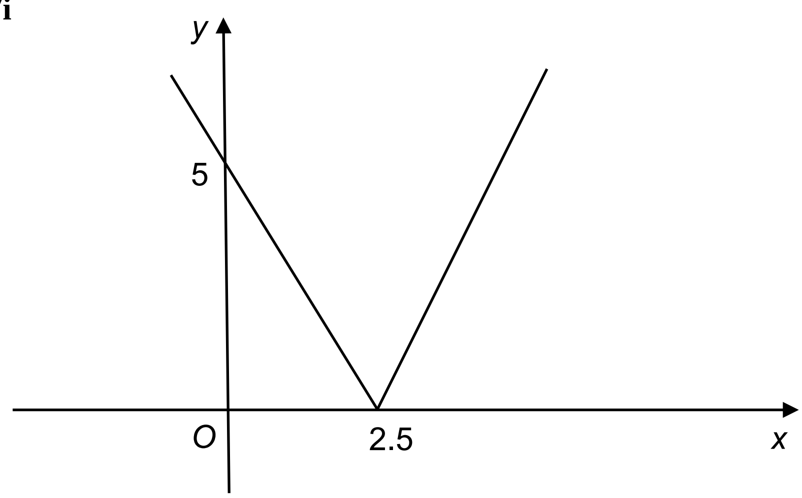
$$A = \frac{1}{2}r^2\left(\sin\theta + \sin 2\theta\right).$$ [3]

(ii) Given that θ can vary, find the value of θ for which A is a maximum. [5]
(You are not required to show that A is a maximum)

Answers 4E/5N Prelim 2020 AMath Paper 1

No.	Answers	No.	Answers
1	$f'(x) = \frac{-19}{(2x-3)^2}$ $\therefore$ Decreasing Function	8i	$p > 2$
2	$m = 0.0405$	8ii	$b^2 - 4ac > 32$ Since $b^2 - 4ac$ is always positive for all real values of k, the line will intersect the curve at 2 distinct pts.
3i	$b = (a+2)^2 + 1$	10i	$a = 3 \quad b = 6$
3ii	$a \geq 1$ or $a \leq -5$ (reject)	10ii	$g(x) = -8\sin^3 x + 6\sin x - 5$
4i	$\frac{dy}{dx} = \frac{3k(x+1)}{\sqrt{2x+3}}$	11i	$t = \frac{3\pi}{4}$
4ii	$k = \frac{3}{4}$	11ii	$\left(\frac{27\pi}{8} - 9\right)$ units^2
		12ii	12 seconds
5i	$P(0, 7)$	12iii	When $v = 0$, $t = 7$ or $t = -6$ (reject) Since the particle is at instantaneous rest when $t = 7$, the particle changes its direction only once.
5ii	$2y = -3x + 1$		
6ii	$x = 45°,\ 114.3°,\ 155.7°$		
7ii	$x = \frac{17}{3}$ or $x = -7$	13ii	$\theta = 0.936$ rad
7iii	$m \geq 5$		

7i



9i