Name:	Register Number:	Class:

4E
5N

BEDOK GREEN SECONDARY SCHOOL

Preliminary Examination 2020

4E
5N

ADDITIONAL MATHEMATICS

4047/01

Paper 1

14 September 2020

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Number of additional writing paper used (if any)	
Number of additional graph paper used (if any)	

For Examiner's Use
80

This document consists of **18** printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(a)** Given that $\tan A = \frac{8}{15}$, $\cos B = \frac{24}{25}$ and A and B are in different quadrants, find, without using a calculator, the value of

(i) $\tan(-B)$, [1]

(ii) $\cos(90° - B)$, [1]

(iii) $\cos\left(\frac{A}{2}\right)$. [2]

1 **(b)** Given that $\dfrac{\cos(A+B)}{\cos(A-B)} = \dfrac{2}{3}$, find the value of $\tan A \tan B$. [3]

2 **(a)** Solve the equation $\left|2x^2 - 15\right| = -x$. [3]

(b) Solve the equation $3^{3x-2} \times 12^{x-1} \div 2^{-2x+1} = 5$. [3]

2 **(c)** The roots of the quadratic equation $x^2 = 3x - 8$ are α and β.

(i) Show that $\alpha^3 = \alpha - 24$. [2]

(ii) Find a quadratic equation whose roots are $\dfrac{\alpha - 1}{\beta}$ and $\dfrac{\beta - 1}{\alpha}$. [5]

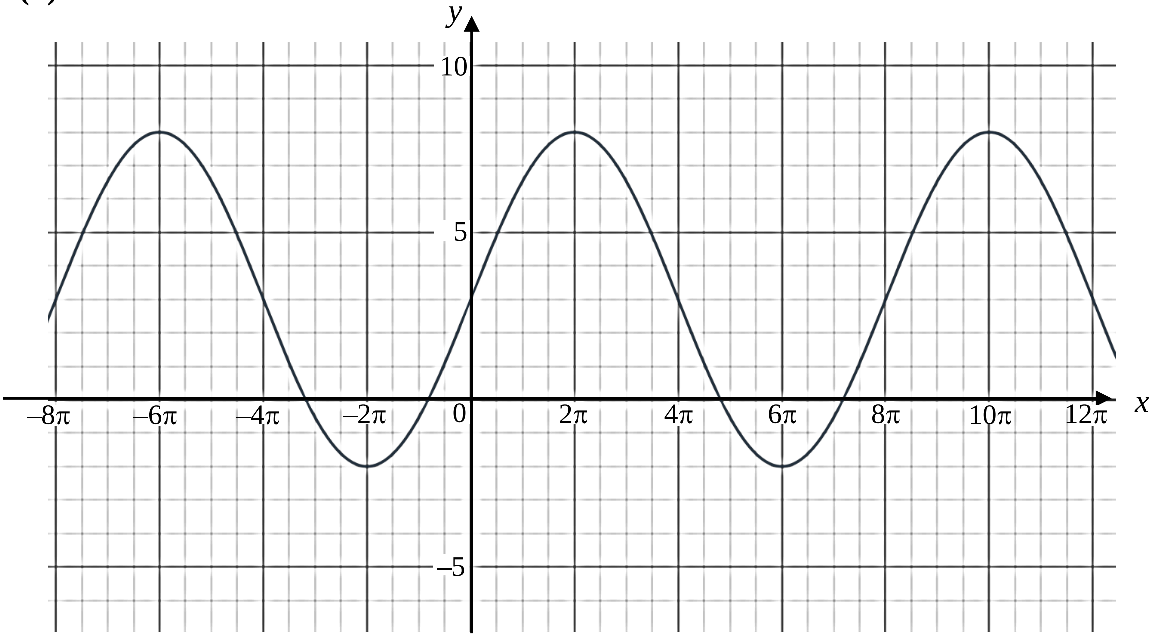
3 It is given that $\int_0^2 f(x)\,dx = 4$ and $\int_2^5 f(x)\,dx = 12$.

(a) Evaluate $\int_0^5 f(x)\,dx$. [1]

(b) Find the value of m for which $\int_0^2 [f(x) + mx^2]\,dx = \int_5^2 f(x)\,dx$. [3]

4 **(a)** State the values between which the principal value of $\sin^{-1} x$ must lie. [1]

(b)



The figure shows part of the graph of $y = a \sin bx + c$.
Find the values of a, b and c. [3]

5 **(i)** Prove that $\frac{1-\sin x\cos x+2\cos^2 x}{\sin^2 x}\equiv 3\cot^2 x-\cot x+1$. [2]

(ii) **Hence** solve the equation $\frac{1-\sin x\cos x+2\cos^2 x}{\sin^2 x}=3$ for $0\le x\le 2\pi$. [4]

6 The function $f(x) = 3x^3 + ax^2 - 10x + b$, where a and b are constants, is exactly divisible by $x - 4$ and leaves a reminder of 5 when divided by $x + 1$.

(a) Find the value of a and of b. [4]

(b) Hence solve f (x) = 0. [2]

7 **(a)** The equation of a curve is $y = 2x^2 - 5x + 13 + k$, where k is a constant.
Find

(i) the set of values of x for which y is increasing, [2]

(ii) the smallest integer value of k for which y is always positive. [3]

(b) Find the range of values of x for which $(2x + 1)(x - 4) \leq x(x - 7)$. [2]

8 A curve has the equation $y = \ln\left(\frac{1-x}{x^2}\right)$, $x < 1$ and $x \neq 0$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) A particle moves along the curve in such a way that the y-coordinate of the particle is changing at a constant rate of 9 units per second.

Find the rate at which the x-coordinate of the particle is increasing at the instant when $y = \ln 2$. [3]

9 The equation of a curve is $y = 3(3x^6 - 1)$.

(i) Find the coordinates of the stationary point on the curve. [3]

(ii) Determine the nature of the stationary point. [1]

10 **(a)** Find $\int \tan^2(3\theta - 2)\, \mathrm{d}\theta$. [2]

(b) Given that $y = \sin 4x + \cos^4 x$, find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{\pi}{4}$. [3]

11 A cuboid has a square base of side $\left(\sqrt{3}-\sqrt{2}\right)\text{m}$ and a volume of $\left(4\sqrt{2}-3\sqrt{3}\right)\text{m}^3$.
Find the height of the cuboid in the form $\left(a\sqrt{2}+b\sqrt{3}\right)\text{m}$, where a and b are integers. [5]

12 The curve has a gradient function $\frac{dy}{dx} = \frac{e^{2x} + a}{e^{2x}}$, where a is a constant.

The gradient of the normal to the curve at the point $(0, -1)$ is $-\frac{1}{2}$.

(a) Find the value of a. [2]

(b) Explain why the curve has no stationary point. [1]

(c) Find the equation of the curve. [2]

13 The population, P, in thousands, of a certain species of animal, has been decreasing each year from 2000 to 2018.

The variables P and t are related by the equation $P = P_0 e^{-\omega t}$, where P_0 and ω are constants and t is the time in years since 1st January 2000. The table gives the values of P and t for some of the years 2006 to 2018.

Start of Year	2006	2009	2012	2015	2018
t years	6	9	12	15	18
P	274	203	151	134	83

It is believed that an error was made in recording one of the values of P.

(i) On the given grid, plot $\ln P$ against t for the given data and draw a straight line graph. [2]

13 **(ii)** Use the graph to estimate the value of P_0 and of ω. Give your answers correct to the nearest hundred and to 1 decimal place respectively. [3]

(iii) Hence determine which value of P in the table is the incorrect recording and use the graph to estimate a value of P to replace the incorrect recording of P. [2]

(iv) Estimate the year that the population of the species of animal will fall below 100 000. [2]

End of Paper