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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2020
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: / INDEX NUMBER:

ADDITIONAL MATHEMATICS 4047/02

Paper 2

11 August 2020

2 hours 30 minutes

0800 – 1030h

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Omission of essential working will result in loss of marks.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **22** printed pages.

Setter: Mdm Mirshasha

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The roots of the quadratic equation $5x-2x^2=2-k$ are α and β. The roots differ by $4\frac{1}{2}$.

(i) Show that $k = 9$. [4]

(ii) Hence find a quadratic equation with integer coefficients whose roots are α^3 and β^3. [4]

2 **(a)** It is given that $\tan(A+B)=8$ and $\tan B=2$. **Without using a calculator**, find the exact value of $\cot A$. [3]

(b) **(i)** Prove that $\sin 2x(\cot x-\tan x)=2\cos 2x$. [3]

(ii) Hence solve the equation $\sin 2x(\cot x - \tan x) = \sec 2x$ for $0 \le x \le \pi$. [4]

3 The equation of circle C_1, with centre A, is $x^2 + y^2 - 8x - 4y + 11 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

(ii) Explain why the line $x = 1$ is a tangent to C_1. [1]

The equation of a second circle C_2, with centre B, is $(x+3)^2+(y-2)^2=R^2$.

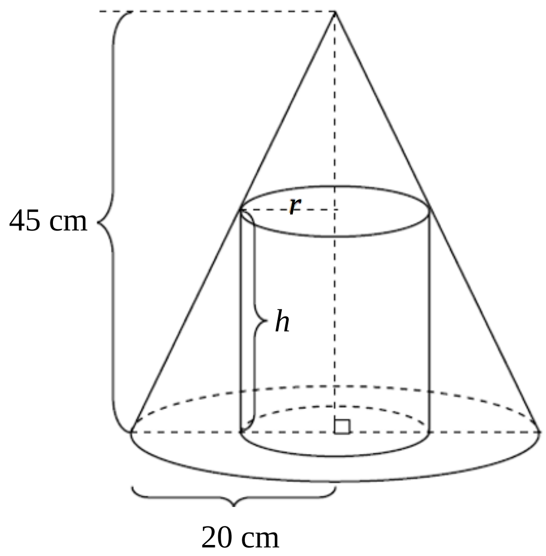
(iii) Find the length of AB. [2]

(iv) State the possible value(s) of R such that C_1 and C_2 touch each other at exactly one point. [2]

4 **(a)** Given that $\log_{\sqrt{2}} x = m$ and $\log_4 y = n$, express $\dfrac{\sqrt{y}}{x^6}$ in terms of m and n. [4]

(b) Solve the equation $\log_2(3x-2) = \log_4\left(x^2+1\right) + \dfrac{2}{\log_{\sqrt{2}} 2}$. [6]

5 The diagram shows a solid cylinder of radius r cm and height h cm inscribed in a hollow cone of height 45 cm and base radius 20 cm. The cylinder rests on the base of the cone and the circumference of the top surface of the cylinder touches the curved surface of the cone.



(i) Show that the volume, V cm^3, of the cylinder is given by $V = 45\pi r^2 - \dfrac{9}{4}\pi r^3$. [3]

(ii) Given that r can vary, find the maximum volume of the cylinder, leaving your answer in terms of π. [4]

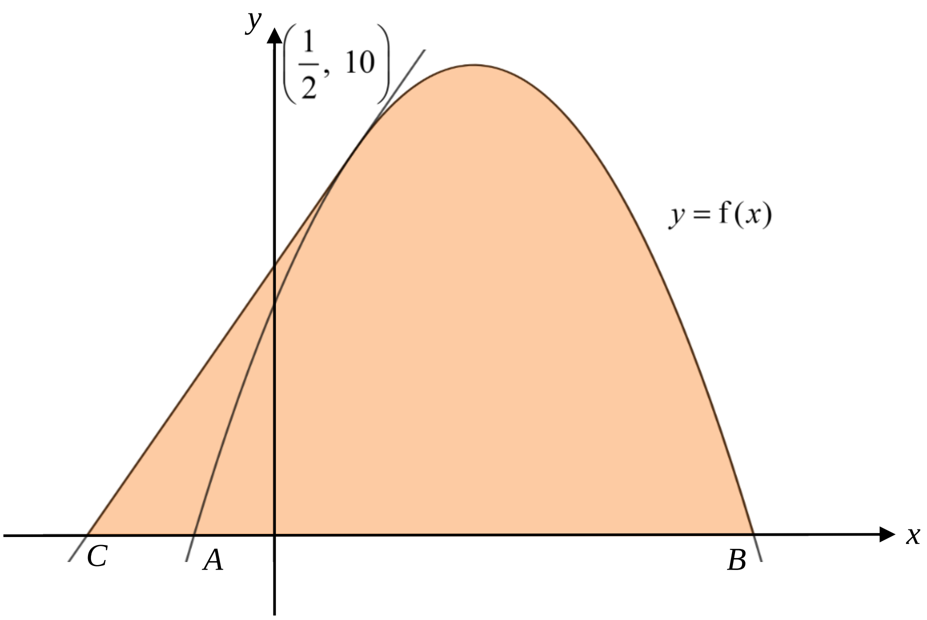
(iii) Hence show that the cylinder occupies at most $\frac{4}{9}$ of the volume of the cone. [2]

6 **(i)** Express $\dfrac{2x^3-20x^2-17x-10}{\left(x^2-4\right)\left(2x^2+1\right)}$ in partial fractions. [5]

(ii) Differentiate $\ln\left(2x^2+1\right)$ with respect to x. [2]

(iii) Using the results from parts **(i)** and **(ii)**, find $\int \frac{2x^3-20x^2-17x-10}{\left(x^2-4\right)\left(2x^2+1\right)}\,dx$. [3]

7



The diagram shows the curve $y = \mathrm{f}(x)$ which intersects the x-axis at points A and B. The tangent to the curve at the point $\left(\frac{1}{2},\ 10\right)$ intersects the x-axis at point C. It is given that $\frac{dy}{dx} = -8x + 10$.

(i) Show that B is the point $(3,\ 0)$ and find the coordinates of C. [5]

(ii) Find the area of the shaded region. [5]

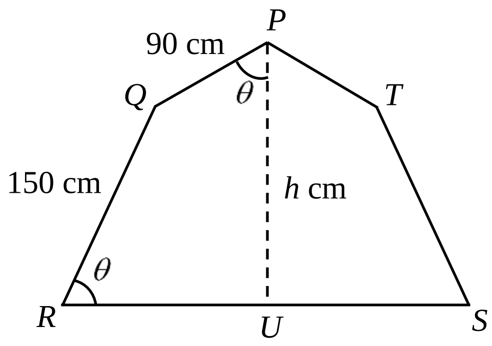
8 **(i)** Sketch the graph of $y=\left|x^2-17x+16\right|$. [3]

(ii) Find the range of values of x for which $\left|x^2-17x+16\right|>54$. [5]

(iii) Given that $\left|x^2 - 17x + 16\right| = k$ has more than 2 distinct solutions, state the range of values of *k*. [2]

(iv) Determine the number of solutions of the equation $\left|x^2 - 17x + 16\right| = 2x - 2$. Justify your answer. [2]

9 The diagram shows the side view *PQRST* of a tent. The tent rests with *RS* on horizontal ground. *PQRST* is symmetrical about the vertical *PU*, where *U* is the midpoint of *RS*. Angle *QPU* = angle *QRU* = θ radians and the lengths of *PQ* and *QR* are 90 cm and 150 cm respectively. The vertical height of *P* from the ground is *h* cm.



(i) Explain clearly why $h = 90\cos\theta + 150\sin\theta$. [2]

(ii) Express *h* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Find the greatest possible value of h and the value of θ at which this occurs. [3]

(iv) Find the values of θ when $h = 160$. [3]

10 A bowl of liquid is heated to a high temperature. It subsequently cools in such a way that its temperature, T°C, is given by $T = 15 + Ae^{-kt}$, where t minutes is the time of cooling and A and k are constants. The table below shows corresponding values of t and T.

t	5	10	15	20	25
T	58.8	40.3	29.6	23.4	19.9

(i) Draw the graph of $\ln(T - 15)$ against t. [3]

(ii) Use the graph to estimate the value of each of the constants A and k. [5]

(iii) State the initial temperature of the liquid. [1]

(iv) Use the graph to estimate the time taken for the temperature of the liquid to drop to half of its original temperature. [2]

End of Paper

Answer Key

1(ii)	$8x^2 - 335x - 343 = 0$	**8(ii)**	$x < -2$, $7 < x < 10$ and $x > 19$
2(a)	$\cot A = \frac{17}{6}$	**8(iii)**	$0 < k \leq 56\frac{1}{4}$
2(bii)	$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$	**8(iv)**	**3**
3(i)	coordinates of A = (4, 2) radius of C_1 = 3 units	**9(ii)**	$h = 30\sqrt{34}\cos(\theta - 1.03)$
3(iii)	7 units	**9(iii)**	Greatest value of h = $30\sqrt{34}$ when θ = 1.03.
3(iv)	4 or 10	**9(iv)**	$\theta = 0.614$ or 1.45
4(a)	2^{n-3m}	**10(ii)**	A =75.6 (accept 73.7 to 77.5) $k = 0.11$ (accept 0.105 to 0.115)
4(b)	$x = \frac{12}{5}$	**10(iii)**	90.6°C (accept 88.7 to 92.5)
5(ii)	$2666\frac{2}{3}\pi\ \text{cm}^3$	**10(iv)**	8.25 minutes (accept 8 to 8.5 minutes)
6(i)	$-\frac{3}{x-2} + \frac{2}{x+2} + \frac{4x}{2x^2+1}$		
6(ii)	$\frac{4x}{2x^2+1}$		
6(iii)	$-3\ln(x-2) + 2\ln(x+2) + \ln(2x^2+1) + c$		
7(i)	Coordinates of C = $(-\frac{7}{6}, 0)$		
7(ii)	31.25 units^2		
8(i)