YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020

CANDIDATE NAME	

CLASS				INDEX NUMBER		

ADDITIONAL MATHEMATICS — 4044/01
4 Normal (Academic)
Paper 1

6 August 2020

Candidates answer on the question paper. **1 hour 45 minutes**

READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

For Examiner's Use
/ 70

This document consists of **16** printed pages.

Setter: Mr Tan Weicun **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Find the value of *m* for which the quadratic equation $mx^2 - 3x + 6 = 0$ has real and equal roots. [3]

[4]

2 Find the rational numbers m and n for which $m\left(3^n\right)=\left(\sqrt{75}\right)^3$, given that $m > 300$. [3]

[5]

3 Factorise $1000x^3 + 27$ and hence express 8027 as a product of two prime numbers. [4]

4 Find the value of x such that $\frac{x^3 + x + 2}{x + 1} = x^2 + 4$, and explain why x cannot be equal to -1. [4]

5 **(i)** By using an appropriate identity, show that the equation $3\sec^2 x = 5(1-\tan x)$ can be simplified into

$$3\tan^2 x + 5\tan x - 2 = 0.$$

[2]

(ii) Find the principal values of x in radians that satisfy the equation

$$3\tan^2 x + 5\tan x - 2 = 0.$$

[3]

6 By using the substitution $u = 2^{\frac{1}{x}}$, or otherwise, solve the equation

$$4^{\frac{1}{x}} - 6\left(2^{\frac{1}{x}}\right) + 8 = 0.$$ [5]

7 **(i)** Differentiate $3\left(x^2 - x + 4\right)^{-2}$ with respect to x. [3]

(ii) Hence, find $\int \frac{14x-7}{\left(x^2 - x + 4\right)^3}\,\mathrm{d}x$. [2]

8 **(i)** Express $15\sin\theta - 8\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R is a positive constant and $0 < \alpha < \frac{\pi}{2}$ radians. [4]

(ii) Hence, find the largest value of $(15\sin\theta - 8\cos\theta)^2$ and the smallest positive θ that corresponds to this value. [2]

9 The line $y = 6 - 2x$ cuts the x-axis and y-axis at points A and B respectively. The perpendicular bisector of line segment AB cuts the point $C\left(-\frac{1}{2}, k\right)$.

(i) Find the value of k. [5]

(ii) Find the area of triangle ABC. [2]

10 Find the coordinates of all the stationary points of the curve $y = x^3 + 3x^2 - \frac{7}{2}$ and determine the nature of each stationary point. [6]

11 A curve passing through $P\left(-1, \frac{5}{2}\right)$ has the gradient function given by $\frac{dy}{dx} = kx^2 - 6x + 3$. The tangent to the curve at P has the equation $y = mx + \frac{1}{2}$.

(i) Find the value of m and of k. [4]

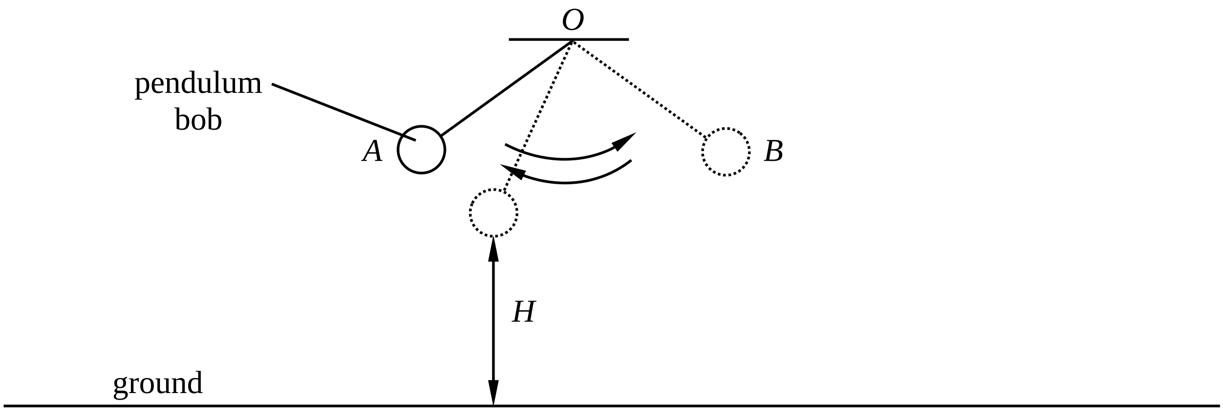
(ii) Hence, find the equation of the curve. [3]

12 The function f, defined by $f(x) = 2x^3 + ax^2 - 2bx - 3$, has a factor $(x+3)$ and leaves a remainder of 4 when divided by $(x+1)$.

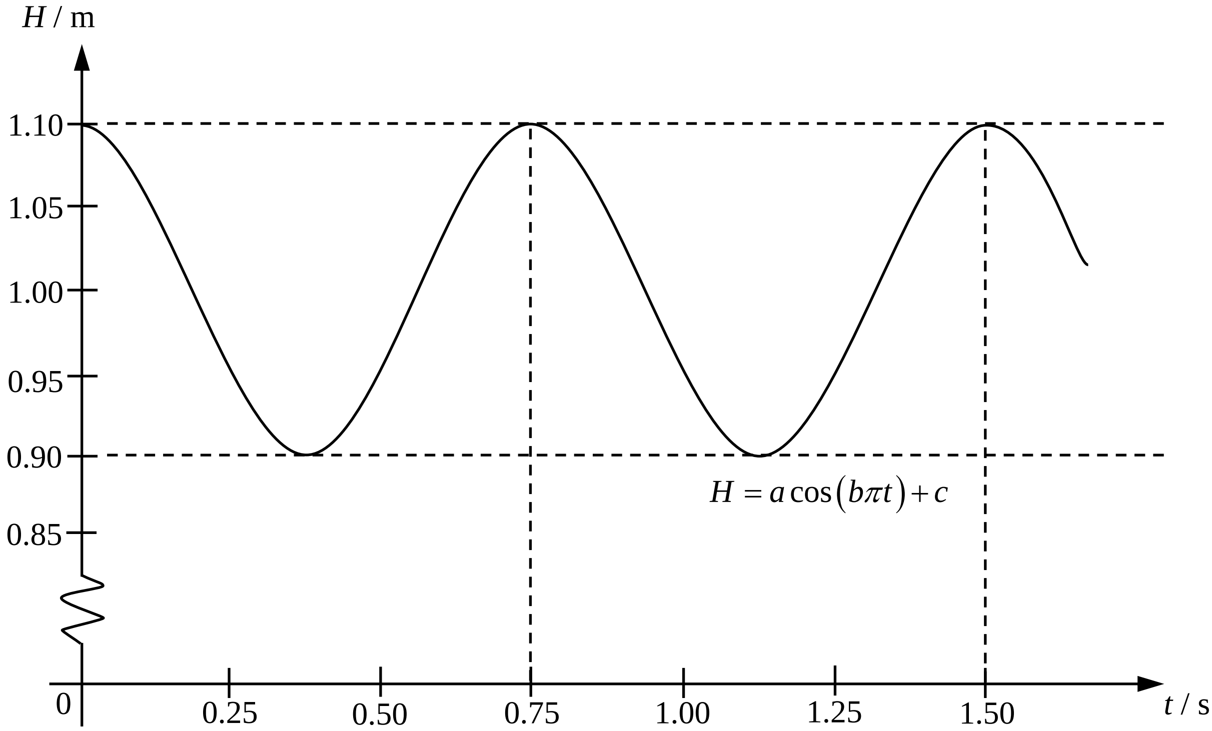
(i) Find the value of *a* and of *b*. [4]

(ii) Hence, solve the equation $f(x) = 0$. [3]

13 The diagram shows a pendulum fixed at point *O*. The pendulum was made to oscillate continuously between *A* and *B*. The vertical height of the pendulum bob from the ground *t* seconds after starting from point *A* is given by *H* m.



It is given that the relationship between *H* and *t* can be modelled by the trigonometric function $H = a\cos(b\pi t) + c$. The graph of *H* against *t* is shown below.



(i) Using the graph of $H = a\cos(b\pi t) + c$ **,**

(a) state the time taken for the pendulum to swing from *A* to *B* once, [1]

(b) find the value of *a*, of *b* and of *c*. [4]

(c) find the **first three** values of *t* when the pendulum bob is nearest to the ground. [2]

(ii) Find the vertical height of the pendulum bob from the ground 10 seconds after it started oscillating from *A*. [1]

End of Paper