Section A: Pure Mathematics [43 m]

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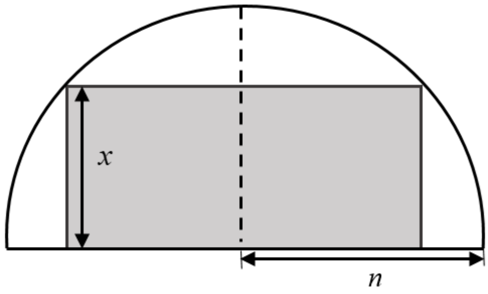

The shaded region in the diagram above shows the cross sectional view of a cylinder open on both ends inscribed in a hemisphere with fixed radius n cm. If the diameter of the cylinder is x cm, show that the surface area A cm^2 of the cylinder is given by

$$A = 2\pi x\sqrt{n^2 - x^2}\ .$$ [2]

Use differentiation to find, in terms of n, the value of x which gives a maximum value of A, justifying that this value gives maximum A. Find also the ratio of the diameter of the cylinder to the height of the cylinder, in this case, simplifying your answer. [5]

2 The points A and B have positions vectors $\mathbf{i}-2\mathbf{j}-\mathbf{k}$ and $s\mathbf{i}+t\mathbf{j}-4\mathbf{k}$ respectively. The plane p has equation $3x-2y-z=5$. Given that the line AB has no points in common with the plane p, express s in terms of t. [2]

Find the shortest distance between the line AB and the plane p. [2]

Another plane q is perpendicular to plane p and contains the line with equation $\dfrac{x-5}{3}=\dfrac{1-y}{5}=\dfrac{1-z}{2}$. Find the equation of plane q in cartesian form. [3]

Find the position vector of the point which is a reflection of A in the line of intersection between p and q. [6]

3 **(a)** The complex numbers w and z are such that $w=-1-\mathrm{i}\sqrt{3}$ and $z^*+z\neq 0$. The points P and Q in the Argand diagram represent the complex numbers w and z^*+z respectively. Find $\arg(w)$, leaving your answer in exact form. Hence, state the possible values of angle POQ, where O is the origin. [3]

(b) Given that z_1 and z_2 are complex numbers such that $\dfrac{3z_1-z_2}{3z_1+z_2}$ is purely imaginary, show that $|z_2|=3|z_1|$. [3]

4 **(a)** **(i)** Find the derivative of e^{x^2+2x}. [1]

(ii) Hence, find $\int (x+1)^3\,\mathrm{e}^{x^2+2x}\ \mathrm{d}x$. [3]

(b) By using the substitution $x=a\sin\theta$, find

$$\int_0^a \sqrt{a^2-x^2}\ \mathrm{d}x.$$ [4]

(c) **(i)** On the same diagram, sketch the graphs of $y=\left|x^2-2\right|$ and $y=x$ for $0\leqslant x\leqslant\sqrt{2}$. [2]

The region R is bounded by the curve $y=\left|x^2-2\right|$, the line $y=x$ and the x-axis.

(ii) Find the exact area of R. [4]

[Turn Over

(iii) Find the exact volume of the solid obtained when region R is rotated through 4 right angles about the y-axis. [3]

Section B: Statistics [57 m]

5 For events A and B, it is given that $\mathrm{P}(A)=0.6$, $\mathrm{P}(A\cup B)=0.7$ and $\mathrm{P}(B\mid A)=0.3$.

(i) Show that $\mathrm{P}(B)=0.28$ and determine with a reason, if A and B are independent. [3]

(ii) An event C is such that A and C are mutually exclusive and $\mathrm{P}(C)>0.25$. Find the range of values of $\mathrm{P}(B'\cap C)$. [3]

6 A group of 8 people consists of 4 boys and 4 girls.

(a) They go to a theatre and sit in the last row of 10 adjacent seats at random.

(i) Find the number of ways in which they can be seated if they are not all seated together. [2]

(ii) Given that they are not all seated together, find the probability that the boys are seated together and the girls are also seated together. [2]

(b) At a dinner, the group sits around a square table which has two seats on each side.

(i) Find the number of ways in which they can be seated if there must be a boy and a girl on each side of the table. [3]

(ii) There are 3 different desserts on the menu and every one selects a dessert each. Find the number of different selections that can be made by the group. [1]

7 In a game at a funfair, a player repeatedly tosses a fair six-sided die. If the toss is an odd number, the player loses and the game ends. Otherwise, the player continues to toss the die until a '6' is shown and the game ends. The score, X, is the number of tosses that it takes for the player to toss the first '6', and is 0 if the player loses the game.

(i) Show that $\mathrm{P}(X=0)=\dfrac{3}{4}$. [2]

(ii) What is the probability that the score is 1 given that the player does not lose? [2]

It is known that for $0<p<1$, $\displaystyle\sum_{r=1}^{\infty} rp^{r-1}=\frac{1}{(1-p)^2}$ and $\displaystyle\sum_{r=1}^{\infty} r^2p^{r-1}=\frac{1+p}{(1-p)^3}$.

(iii) Show that $\mathrm{E}(X)=\dfrac{3}{8}$. Find $\mathrm{Var}(X)$. [4]

8 A company sells bags of cement in two sizes. The mass of a large bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg. The mass of a small bag of cement can be modelled by a normal distribution with mean 30 kg and standard deviation 1.5 kg.

(i) Find the mass that is exceeded by 99% of the large bags. [2]

(ii) Find the probability that total mass of three small bags is less than twice the mass of one large bag. [3]

(iii) State an assumption needed for your calculation in part **(ii)**. [1]

Three large bags are selected at random.

(iv) Find the probability that two weigh more than 53 kg and one weighs less than 53kg. [3]

9 A company sells a certain type of fitness tracker batteries. The company claims that the mean lifetime of the batteries is at least 2000 hours. The production manager of the company wishes to take a random sample of batteries from the thousands produced in a day at his factory, for quality control purposes.

(i) State what it means for a sample to be random in this context. [1]

A random sample of 200 batteries is taken and it is found that the mean lifetime was 1995 hours and the standard deviation was 25.5 hours.

(ii) Test, at the 1% significance level, whether there is any evidence to doubt the company's claim. [6]

(iii) State two assumptions or approximations that are involved when carrying out the significance test using the above sample data. [2]

The company wishes to test whether the average lifetime of a second type of battery is longer than 2000 hours. It can be assumed that the lifetime of the second type of battery has a normal distribution with variance σ^2. 8 randomly chosen batteries of the second type are tested and it is found that their lifetimes, in hours, are as follows.

2001.5 1997.3 1989.0 2018.7 2030.7 2023.0 2027.5 2013.3

The company carries out a hypothesis test at the 5% level of significance.

(iv) Explain, with justification, how the population variance of the lifetimes, σ^2, will affect the conclusion. [4]

10 At a clinic, records show that 100p%, where $0 < p \leqslant 0.5$, of patients who make an afternoon appointment fail to turn up. The clinic has one doctor and he sees patients by appointment only. During the opening hours in the afternoon, the doctor has time to see only 15 patients. On any afternoon, a fixed number of patients is accepted to make an afternoon appointment.

(a) State, in context, two assumptions needed for the number of patients who fail to turn up in an afternoon after making an appointment to be well modelled by a binomial distribution. [2]

Assume now that these assumptions do in fact hold.

(b) On a particular afternoon, 15 patients made an appointment to see the doctor.

(i) It is known that there is a probability of 0.02 that 6 or 7 patients fail to turn up for the afternoon appointment. Write down an equation for the value of p, and find this value numerically. [2]

Given that $p = 0.2$,

(ii) find the probability that all 15 patients turn up, [1]

(iii) find the probability that at least 3 but not more than 5 patients fail to turn up for the afternoon appointment, [2]

(iv) find the probability that more than 100 patients fail to turn up for the afternoon appointment in a month. (You may assume that there are 30 afternoons in a month and the clinic is open on all 30 afternoons.) [3]

(c) To improve efficiency, the doctor decides to accept more than 15 appointments in the afternoon. However, he is still able to see only 15 patients and $p = 0.2$. Find the largest number of afternoon appointments that the doctor should accept if he wants to have at least a 90% chance of seeing all the patients who turn up. [3]