



TAMPINES MERIDIAN JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

CANDIDATE
NAME

CIVICS
GROUP

H2 MATHEMATICS

9758/02

Paper 2

21 September 2020

3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or
1 decimal place in the case of angles in degrees, unless a different
level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless
a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not
allowed in a question, you are required to present the mathematical
steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your
answers.

The number of marks is given in brackets [] at the end of each
question or part question.

The total number of marks for this paper is 100.

For Examiners' Use	
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This document consists of 28 printed pages and 0 blank page.



Section A: Pure Mathematics [40 marks]

1 Do not use a calculator in answering this question.

The equation $az^3 + bz^2 + cz + d = 0$ has a root $z = ki$, where k is real and non-zero. It is given that the coefficients a, b, c and d are real where a, b are non-zero.

- (i) Show that $ad = bc$. [2]
- (ii) Verify that the condition in part (i) is satisfied for the equation $z^3 - 9z^2 + 9z - 81 = 0$. [1]
- (iii) Hence, or otherwise, find the two roots, z_1 and z_2 , of $z^3 - 9z^2 + 9z - 81 = 0$ which are of the form $z = ki$, where k is real. Deduce that the third root $z_3 = 9$. [3]
- (iv) Show z_1, z_2 and z_3 on an Argand diagram. [2]

2 A curve C has parametric equations

$$x = 1 + e^t, \quad y = t - e^t \quad \text{for } t > 0.$$

- (i) Sketch the graph of C . [2]
- (ii) Find the equation of the tangent to C at the point $P(1 + e^p, p - e^p)$ in the form $y = ax + b$, where a and b are in terms of p . [3]
- (iii) The tangent to C at P meets the y -axis at the point Q . Point R lies on another curve such that P is the midpoint of QR . Find parametric equations of the curve traced by R as p varies. [4]

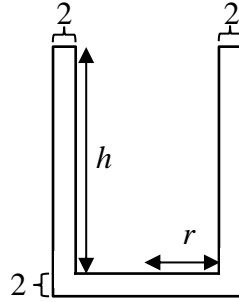
3 It is given that

$$f(x) = \begin{cases} 1, & \text{for } -1 < x \leq 0, \\ \frac{1}{1+4x^2}, & \text{for } 0 < x \leq 1, \end{cases}$$

and that $f(x) = f(x+2)$ for all real values of x .

- (i) Find the exact value of $f(23)$. [2]
- (ii) Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$. [3]
- (iii) Find $\int_{-2}^1 f(x) dx$, leaving your answer in exact form. [4]
- (iv) The region bounded by the curve $y = f(x)$, the line $y = \frac{1}{5}$ and the y -axis is rotated through 2π radians about the y -axis. Find the volume of the solid generated, giving your answer to three decimal places. [2]

4



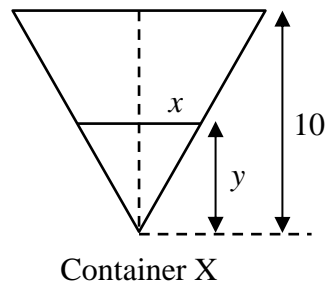
The diagram shows the cross-sectional view of an open cylindrical tank with a capacity of 300 m^3 . The base and the walls of the tank are made of a special material with a thickness of 2 m . The internal radius and internal height of the tank are $r \text{ m}$ and $h \text{ m}$ respectively.

- (i) Show that the volume of the special material used to make the tank, $V \text{ m}^3$, is given

$$\text{by } V = \pi(r+2)^2 \left(\frac{300}{\pi r^2} + 2 \right) - 300. \quad [2]$$

- (ii) Use differentiation to find the exact value of r when V is a minimum. [5]

(iii)



The cylindrical tank is filled to the brim with acid. The acid is then drained from the cylindrical tank into a container X of negligible thickness in the shape of a vertically inverted cone as shown in the diagram at a rate of 0.1 m^3 per minute. When the acid is completely drained from the cylindrical tank, the depth of the acid in container X is 10 m . The radius of the acid surface and depth of the acid in container X at time t minutes, are $x \text{ m}$ and $y \text{ m}$ respectively. Find the rate of change of y when $y = 5$. [5]

[It is given that the volume of a circular cone with base radius r and height h is

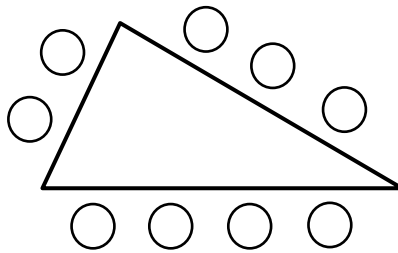
$$\frac{1}{3} \pi r^2 h.]$$

Section B: Probability and Statistics [60 marks]

- 5 A random variable X has the probability distribution given in the following table.

x	4	5	6	7	8
$P(X = x)$	$\frac{1}{10}$	p	q	$\frac{1}{5}$	$\frac{3}{10}$

- (i) Given that $E(X) = 6.5$, find p and q . [4]
- (ii) Find $E(|X - 6|)$. [1]
- 6 At Lalaland pre-school, Josh and his eight other classmates sit around a triangular table as shown in the diagram below.



- (i) Find the number of ways to seat the 9 students such that
- (a) there are no restrictions, [1]
- (b) Josh does not sit next to one particular student on the same side of the triangular table. [2]
- (ii) During music lesson, Josh and his classmates are to stand in a circle. Find the number of different possible arrangements if Josh wants to be adjacent to at least one of his good friends, Eden and Farhan. [3]
- 7 For events A , B , and C , it is given that $P(A) = \frac{2}{7}$, $P(A \cup B) = \frac{5}{9}$, $P(A \cap B') = \frac{1}{8}$, and $P(C) = \frac{4}{5}$. Events B and C are found to be independent.
- (i) Find $P(A|B')$. [3]
- (ii) Determine, with a reason, if events A and B are independent. [1]
- (iii) Find $P(B' \cap C)$. [2]
- (iv) Hence find the smallest possible value of $P(A \cap B' \cap C)$. [2]

8 Finger puppets of characters from a movie are packed individually in opaque capsules to be sold. On average, one in twelve capsules contains the main character. An avid fan buys n capsules and the number of capsules that contain the main character is the random variable F .

- (i) State, in context, two assumptions needed for F to be well modelled by a binomial distribution. [2]

Assume now that F has a binomial distribution.

- (ii) Given that the probability that the avid fan gets at least 2 main characters exceeds 0.9, find the least value of n . [2]

The capsules are packed and sold independently in boxes of ten.

- (iii) Five boxes are randomly chosen. Find the probability that two boxes each contains exactly 1 capsule with the main character and three boxes each contains at least two capsules with the main character. [3]

- 9** (i) X and Y are independent random variables with the distributions $N(\mu, 5^2)$ and $N(12, \sigma^2)$ respectively. Given that $P(X + Y < 12) = 0.25$ and $P(Y - X > 16) = 0.085$, show that $\mu = 3.9$ correct to 1 decimal place, and calculate the value of σ . [6]
- (ii) The random variable K is the sum of n independent observations of X . State the approximate value of $P(K > 3n)$ as n becomes very large, justifying your answer. [2]

10 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

A visit to Marvelous Dental Clinic for cleaning and polishing teeth involves registration followed by treatment. The registration time follows a normal distribution with mean 6.3 minutes and standard deviation 1.4 minutes while the treatment time follows an independent normal distribution with mean 15.1 minutes and standard deviation 1.2 minutes. It can be assumed that the time taken to register and treat all patients are independent of one another.

- (i) Find the probability that the total registration time of two randomly chosen patients is less than twice the treatment time of another randomly chosen patient by at least 10 minutes. [3]
- (ii) There is a probability of at most 0.99 that the average treatment time of n randomly chosen patients is more than 14 minutes. Find the largest possible value of n . [3]
- (iii) A new process is introduced to improve the efficiency of the service rendered at Marvelous Dental Clinic such that the registration time is reduced by 10% and the treatment time is reduced by 5%. Find the probability that the total time taken to register and treat 8 randomly chosen patients is at most 150 minutes. [3]

It is found that the treatment time at Wonderful Dental Clinic has mean 15 minutes and standard deviation 10 minutes.

- (iv) Explain why the treatment time in Wonderful Dental Clinic is unlikely to be normally distributed. [1]
- (v) Find the probability that the average treatment time of 50 randomly chosen patients at Wonderful Dental Clinic is less than 12 minutes. [3]

- 11** Supplier A claims that their bananas have a mass of 125 g on average. A supermarket decides to test if Supplier A's claim is valid. A random sample of 30 bananas is taken from Supplier A and the masses are measured. The unbiased estimates for the population mean and variance are 121 g and 102.01 g^2 respectively.
- (i) Test, at the 2% significance level, whether Supplier A's claim is valid. [5]
 - (ii) State the meaning of this p -value in context. [1]
 - (iii) Supplier A's bananas are also advertised as having potassium content of 420 mg per 100 g. In a test at the 5% significance level, it is found that there is significant evidence that the population mean potassium content of bananas from Supplier A is overstated. Using only this information, and giving a reason in each case, state whether each of the following statements is necessarily true, necessarily false, or neither necessarily true nor necessarily false.
 - (a) There is significant evidence at the 8% significance level that the population mean potassium content is less than 420 mg per 100 g. [2]
 - (b) There is significant evidence at the 5% significance level that the population mean potassium content is not 420 mg per 100 g. [2]
 - (iv) Supplier B claims that their bananas have a mean mass of more than 125 g. Supplier B's bananas are normally distributed and the population standard deviation is 10 g. The supermarket decides to test the claim and measures the masses of a random sample of 20 bananas from Supplier B.
Given that Supplier B's claim is valid at the 5% level of significance, find the range of possible values of the sample mean. [3]

End of Paper