Name:	Register Number:	Class:

4E
5N

BEDOK GREEN SECONDARY SCHOOL

Preliminary Examination 2020

4E
5N

ADDITIONAL MATHEMATICS

4047/02

Paper 2

15 September 2020

2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Number of additional writing paper used (if any)	

For Examiner's Use
100

This document consists of **21** printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mu \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mu \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(a)** **(i)** Given that $u = 3^x$, express $3^{2x} - 3^{x+2} = 3^x - 9$ as an equation in *u*. [1]

(ii) Hence find the values of *x* for which $3^{2x} - 3^{x+2} = 3^x - 9$. [3]

(iii) Explain why the equation $3^{2x} - 3^{x+2} = 3^x - k$ has no solution when $k > 25$. [2]

1 **(b)** Given that lg $x = a$ and lg $y = b$, express $\lg\sqrt{\dfrac{1000y}{x^2}}$ in terms of a and of b. [2]

(c) Solve the equation $\log_4 p - 8\log_p 4 = 2$. [5]

(d) State the range of values of x for $\log_{x-1}(6-x)$ to be defined. [1]

2 **(i)** Given that $y = \dfrac{3e^{2x}}{2x+1}$, find the value of k for which $\dfrac{dy}{dx} = \dfrac{kxe^{2x}}{(2x+1)^2}$. [3]

(ii) Hence evaluate $\displaystyle\int_0^3 \frac{8xe^{2x}}{(2x+1)^2}\,dx$. [2]

3 **(i)** Express $y = -x^2 + 4x - 3$ in the form $y = a - (x+b)^2$. [2]

(ii) Hence sketch the graph of $y = |-x^2 + 4x - 3|$, giving the coordinates of the turning point and the y-intercept. [2]

(iii) On the same diagram, sketch the graph of $y = x^{\frac{3}{4}}$, $x \geq 0$. [1]

(iv) State the number of solutions to the equation $\left|-x^2 + 4x - 3\right| = \sqrt[4]{x^3}$. [1]

4 **(i)** By considering the general term in the binomial expansion of $\left(x+\frac{2a}{x}\right)^6$, where a is a constant, explain why there are no odd powers of x in this expansion. [2]

(ii) Write down, and simplify, the first three terms in the expansion of $\left(x+\frac{2a}{x}\right)^6$ in descending powers of x, where a is a constant. [2]

4 **(iii)** The first three terms in the expansion of $(b+x^2)\left(x+\frac{2a}{x}\right)^6$ in descending powers of x are $x^8+25x^6+264x^4$, where a and b are integers.

Find the value of a and of b. [5]

5 **(i)** Express $\dfrac{4x^3+12x^2+3x+1}{x(2x+1)^2}$ in partial fractions. [6]

(ii) Hence find $\displaystyle\int \frac{4x^3+12x^2+3x+1}{x(2x+1)^2}\,\mathrm{d}x$. [2]

6 The equation of a circle, C_1, with centre A, is $x^2 + y^2 + 6x - 8y + 9 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

(ii) Show that the point M (1, 4) lies on C_1. [1]

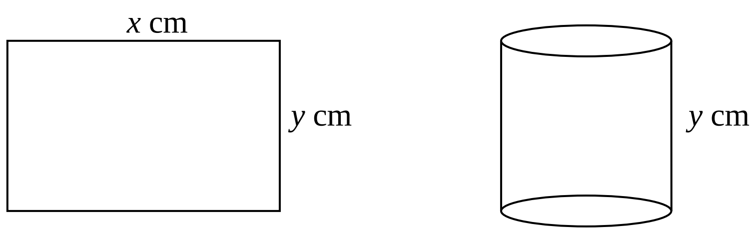
(iii) Find the equation of the tangent to C_1 at M. [1]

6 **(iv)** Find the equation of another circle C_2 for which AM is the diameter. [3]

(v) The line $y = x + k$, where k is a constant, is a tangent to C_2.

Find the possible exact values of k. [4]

7 The diagram shows a rectangular metal plate of length x cm and width y cm.
The metal plate is rolled into the shape of a cylinder of height y cm.



The perimeter of the metal plate is 25 cm.

(i) Express y in terms of x. [1]

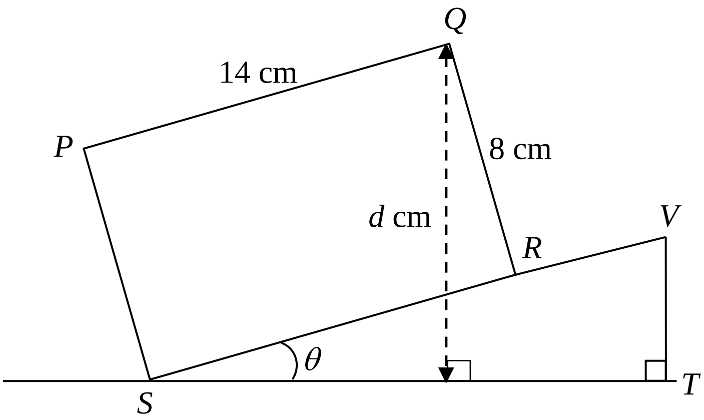
(ii) Show that the volume, V cm^3, of the cylinder is given by $V = \frac{x^2}{8\pi}(25 - 2x)$. [3]

7 **(iii)** Given that x can vary, find the value of x for which V is either a maximum or a minimum. [3]

(iv) Find this stationary value of V, correct to 3 significant figures. [1]

(v) Determine whether this value of x makes V a maximum or a minimum. [2]

8



The diagram shows the frontal view of a rectangular block *PQRS*, with dimensions 14 cm by 8 cm. The block is placed on an adjustable ramp *VS* such that it is tilted at an acute angle θ and $\angle VTS = 90°$. The ramp is placed on a horizontal surface *ST* and the perpendicular distance from *Q* to *ST* is *d* cm.

(i) Show that $d = 8\cos\theta + 14\sin\theta$. [3]

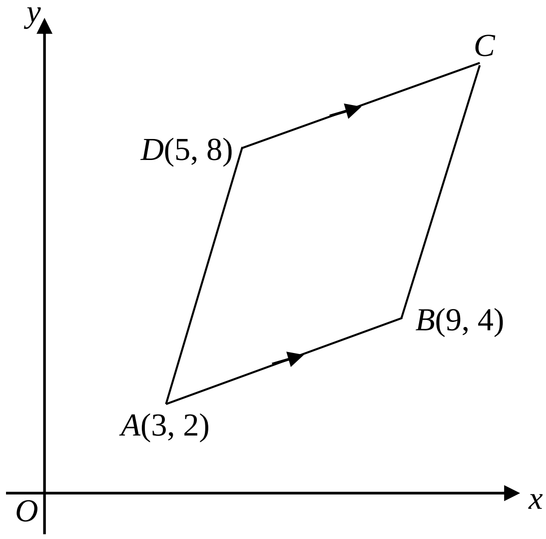
(ii) Express *d* in the form $R\sin(\theta + \alpha)$, where *R* > 0 and α is acute. [2]

8 **(iii)** The perpendicular distance from Q to ST is $\sqrt{200}$ cm.

Find the smallest angle θ. [3]

(iv) State the maximum value of d. [1]

9 The diagram shows a quadrilateral with vertices $A(3, 2)$, $B(9, 4)$, C and $D(5, 8)$.
The sides AB and DC are parallel.



(a) Find the equation of DC. [2]

(b) Find the equation of the perpendicular bisector of BD. [3]

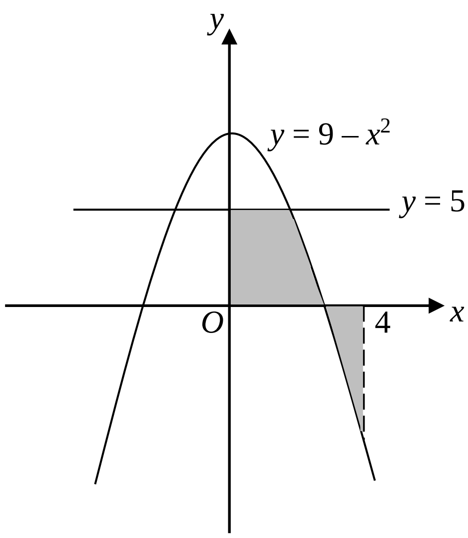
9 **(c)** The coordinates of the point *C* is equidistant from *B* and *D*.

Find the coordinates of *C*. [3]

(d) Find the area of the quadrilateral *ABCD*. [2]

10 The graph shows the curve $y = 9 - x^2$ and the line $y = 5$.

Find the area of the shaded region. [5]



11 A particle P leaves a fixed point O and moves in a straight line so that, t s after leaving O, its velocity, V m/s, is given by $v = -3t^2 + kt + 15$, where k is a constant.

(a) Given that its deceleration is 8 m/s^2 when $t = 2$, find the value of k. [2]

(b) Find the value of t for which the particle is at instantaneous rest. [2]

11 **(c)** Find the maximum velocity of the particle. [2]

(d) Find the time taken for the particle to return to point *O*. [3]

11 **(e)** Find the average speed of the particle in the first 4 seconds. [3]

End of Paper