VICTORIA JUNIOR COLLEGE

JC 2 PRELIMINARY EXAMINATION 2022

CANDIDATE NAME	

CLASS		INDEX NUMBER	

H2 MATHEMATICS **9758/02**
Paper 2 **3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing paper

READ THESE INSTRUCTIONS FIRST

Write your class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

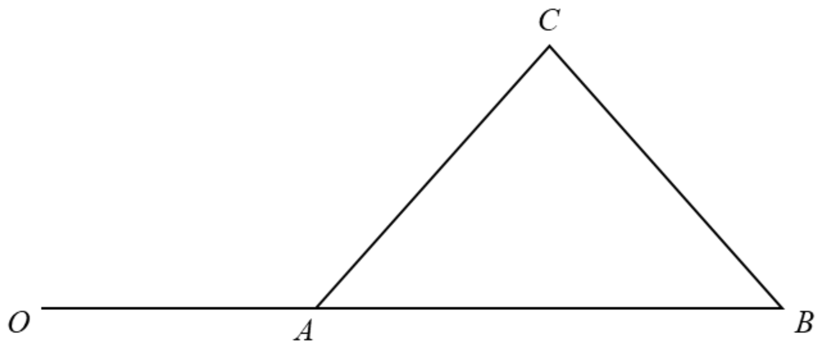
Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **21** printed pages and **3** blank pages.

Section A: Pure Mathematics [40 marks]

1



Referred to the origin O, points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.
It is also given that OAB is a straight line (see diagram).

(i) Show that the area of triangle ABC can be written in the form $k\left|(\mathbf{b}-\mathbf{a})\times\mathbf{c}\right|$, where k is a constant to be determined. [3]

It is given that $\overrightarrow{AB}$ is a unit vector and C is equidistant from A and B.

(ii) Give a geometrical interpretation of $\left|(\mathbf{a}-\mathbf{b})\times\mathbf{c}\right|$. [1]

(iii) Show that OB has length $\left|\mathbf{c}.\mathbf{b}-\mathbf{c}.\mathbf{a}\right|+q$, where q is a constant to be determined. [3]

2 **(a)** Functions f and g are defined by

$$\mathrm{f}: x \mapsto 2 - \mathrm{e}^{x+a}, \quad \text{for } x \in \mathbb{R},\ x > -2,$$

$$\mathrm{g}: x \mapsto x^2 + 2x, \quad \text{for } x \in \mathbb{R},\ x < -1,$$

where a is a constant.

(i) Find $\mathrm{g}^{-1}(x)$. [2]

(ii) Explain why the composite function fg exists. [2]

(iii) Find, in terms of a, an expression for $\mathrm{fg}(x)$ and write down the domain of fg. [2]

(iv) Find the range of fg, giving your answer in terms of e and a. [2]

(b) The function h is defined by $\mathrm{h}: x \mapsto \mathrm{e}^{|x+\lambda|}, \quad x \in \mathbb{R}$, where λ is a constant.
Does h have an inverse? Justify your answer. [2]

3 **(i)** Show that $\cos\left(x+\frac{\pi}{6}\right)+\cos\left(x-\frac{\pi}{3}\right)=\sqrt{2}\cos\left(x-\frac{\pi}{12}\right)$. [2]

At time t seconds after turning on a switch, the total potential difference across two alternating current power supplies, V, is given by $\mathrm{Re}(z_1+z_2)$, where

$$z_1 = 2\mathrm{e}^{\left(t+\frac{\pi}{6}\right)\mathrm{i}} \text{ and } z_2 = 2\mathrm{e}^{\left(t-\frac{\pi}{3}\right)\mathrm{i}}.$$

(ii) Express z_1+z_2 in the form $r\mathrm{e}^{(t-\alpha)\mathrm{i}}$, where $r>0$ and $-\pi<\alpha\leq\pi$, leaving your values of r and α in exact form. [4]

(iii) From the time the switch is turned on, find the amount of time it takes for V to be first at half its maximum value, giving your answer in seconds, correct to 3 decimal places. [2]

(iv) The engineer modified the power supplies so that $z_1 = z_2 = w^{2n}e^{it}$, where $w = 1 + i$ and n is an integer. Show that $V = 2^{n+1}\cos\left(\dfrac{n\pi}{2} + t\right)$. [3]

4 It is given that $\sum_{r=1}^{N} \ln\left[\frac{(r+1)(r+3)}{r(r+2)}\right] = \ln\left[\frac{(N+1)(N+3)}{2}\right]$.

Use this result to find $\sum_{r=4}^{k+2} \ln\left[\frac{r(r+2)}{3(r-1)(r+1)}\right]$, expressing your answer in the form $\ln\left[\frac{(k+2)(k+4)}{a\left(b^{k-1}\right)}\right]$ where a and b are positive integers to be determined. [5]

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A sequence of positive real numbers $v_1, v_2, v_3, \ldots$ is given by $v_1 = 5$ and

$$v_{n+1} = v_n + \sum_{r=1}^{n} \left[(2r+1) + \ln\left[\frac{(r+1)(r+3)}{r(r+2)} \right] \right].$$

Show that $v_{n+1} - v_n = n(n+2) + \ln\left[\frac{(n+1)(n+3)}{2} \right]$. [3]

By considering $\sum_{n=1}^{99} (v_{n+1} - v_n)$, find the numerical value of v_{100}, correct your answer to the nearest integer. [4]

Section B: Probability and Statistics [60 marks]

5 For events A and B, it is given that $\mathrm{P}(A)=0.6$, $\mathrm{P}(B)=0.2$ and $\mathrm{P}(A \mid B')=0.55$. Find

(i) $\mathrm{P}(A \cap B')$, [1]

(ii) $\mathrm{P}(A' \cap B')$. [2]

For a third event C, it is given that $\mathrm{P}(C)=0.4$, $\mathrm{P}(A \cap C)=\mathrm{P}(B \cap C)$, $\mathrm{P}(A' \cap B' \cap C)=0.24$ and $\mathrm{P}(A \cap B \cap C)=0.1$. Determine whether A and C are independent. [3]

6 Four-figure numbers are to be formed from the digits 3, 4, 5, 6, 7 and 8. Find the number of different four-figure numbers that can be formed if

(i) no digit may appear more than once in the number, [1]

(ii) there is at least one repeated digit, but no digit appears more than twice in the number, [3]

(iii) no digit may appear more than once in the number and the sum of all the digits in the number is not divisible by six. [3]

7 An unbiased yellow cubical die has two faces labelled 10, two faces labelled 30 and two faces labelled 50. An unbiased green cubical die has four faces labelled 60, one face labelled 80 and one face labelled 100.

(i) When both dice are thrown, the random variable X is half of the difference between the score on the green die and the score on the yellow die. Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [4]

(ii) Suppose now the yellow die is replaced by an unbiased blue cubical die with two faces labelled 15, two faces labelled 35 and two faces labelled 55. The random variable W is half of the difference between the score on the green die and the score on the blue die. Without doing further calculation, comment on $\mathrm{E}(W)$ and $\mathrm{Var}(W)$ in relation to $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ respectively. [2]

8 An agricultural experiment was carried out to study the effect of a certain fertilizer on the growth of seedlings. The fertilizer is applied at various concentrations to a random sample of ten plots of land. Seeds are sown and two weeks later, the mean height of the seedlings on each plot of land is measured. The results are shown in the table.

Concentration of fertilizer (x grams/m^2)	5	10	20	30	40	50	60	70	80	90
Mean height of seedlings (y cm)	4.2	9.0	15.6	18.5	19.2	22.5	24.0	25.4	25.4	26.2

(i) Draw the scatter diagram for these values, labelling the axes clearly. [1]

It is thought that the mean height of seedlings y can be modelled by one of the formulae

$$y = a + bx \quad \text{or} \quad y = c + d\ln x,$$

where a, b, c and d are constants.

(ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(a) x and y,

(b) $\ln x$ and y. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = a + bx$ or $y = c + d\ln x$ is the better model. [1]

It is required to estimate the value of x for which $y = 20.0$.

(iv) Explain why neither the regression line of x on y nor the regression line of $\ln x$ on y should be used. [1]

(v) Find the equation of a suitable regression line and use it to find the required estimate. [2]

(vi) Re-write your equation from part **(v)** so that it can be used when y, the mean height of seedlings, is given in mm. [1]

9 In each batch of manufactured articles, 5% of the articles are found to be defective. A quality inspection is carried out by checking samples of 20 articles.

(i) If 2 or fewer defective articles are found in the sample of 20, the batch is accepted. Find the probability that the batch is accepted. [1]

(ii) Find the least value of n such that the probability of having less than n defective articles in a sample of 20 articles is greater than 0.99. [2]

(iii) Fifty random samples of 20 articles each were taken. Find the probability that the average number of defective articles per sample is at most 1.25. [3]

(iv) It is proposed that a smaller sample be taken for inspection. Find the largest value of k such that the probability of having at least 1 defective article in a sample of k articles is to be less than 0.4? [3]

10 In this question, you should state clearly the values of the parameters of any normal distribution you use.

A meat supplier imports frozen chickens and frozen ducks which are priced by weight. The masses, in kg, of frozen chickens and frozen ducks are modelled as having normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Frozen chickens	1.5	0.3
Frozen ducks	2.6	0.5

(i) Find the probability that a randomly chosen frozen duck has a mass which is more than twice that of a randomly chosen frozen chicken. [3]

The frozen chickens are imported at a cost price of \$6 per kg and the frozen ducks at \$8 per kg.

(ii) The meat supplier orders 100 frozen chickens and 50 frozen ducks in a particular consignment. Find the probability that the meat supplier paid no more than \$2000 for this consignment. State an assumption needed for your calculation. [5]

(iii) A restaurant owner buys frozen ducks from this supplier, who sells the frozen ducks at a profit of 25%. The restaurant owner does not wish to purchase frozen ducks that are too big or too small. If there is a probability of 0.7 that the restaurant owner paid between \$20.00 and \$$a$ for a randomly chosen frozen duck, find the value of a. [4]

11 (a) The average time taken by George to swim 100 m freestyle is 120.05 seconds. He bought a new pair of special goggles from a salesperson who claimed that the goggles will help George swim faster. After wearing the goggles, the time, t seconds, for George to swim 100 m freestyle of each of 50 randomly chosen timings is recorded. The results are summarised as follows.

$$\sum(t-120.05)=-66.4,\quad \sum(t-120.05)^2=1831.945.$$

(i) Test, at the 10% level of significance, whether the goggles helped George to swim faster. You should state your hypotheses and define any symbols you use. [6]

(ii) Explain why this test would be inappropriate if George had taken a random sample of 10 of his 100 m freestyle timings. [1]

(iii) Explain, in the context of the question, the meaning of a "10% significance level". [1]

(b) The random variable X has distribution $\mathrm{N}\left(\mu,\ \sigma^2\right)$.

A random sample of n observations of X is taken, where n is sufficiently large. The mean and variance of this sample is k and 9 respectively.

(i) A test at the 1% level of significance level indicates that the null hypothesis $\mu = 25$ is rejected in favour of the alternative hypothesis $\mu \neq 25$. Find, in terms of n , the range of values of k , giving non exact answers correct to 4 decimal places. [3]

(ii) Hence state the conclusion of the hypothesis test in the case where $k = 24$ and $n = 42$. [1]

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