Section A: Pure Mathematics [40 marks]

1 On the same axes, sketch the graphs of $y = 2(x-a)^2$ and $y = 3a|x-a|$, where a is a positive constant, showing clearly all axial intercepts. [2]

(i) Solve the inequality $2(x-a)^2 \geq 3a|x-a|$. [4]

(ii) Hence solve $2\left(x-\dfrac{a}{2}\right)^2 \geq 3a\left|x-\dfrac{a}{2}\right|$. [2]

2 It is given that $y = \dfrac{e^{\sin x}}{\sqrt{1+2x}}$.

(i) Show that $\dfrac{1}{y}\dfrac{dy}{dx} + \dfrac{1}{1+2x} = \cos x$. [2]

(ii) By further differentiation of the result in part **(i)**, find the Maclaurin series for y in ascending powers of x, up to and including the term in x^3. [5]

(iii) Use your result from part **(ii)** to approximate the value of $\int_0^1 \dfrac{e^{\sin x}}{\sqrt{1+2x}}\,dx$. Explain why this approximation obtained is not good. [2]

(iv) Deduce the Maclaurin series for $\dfrac{1}{e^{\sin x}\sqrt{1-2x}}$ in ascending powers of x, up to and including the term in x^3. [1]

3 The complex numbers p and q are given by $\dfrac{a}{1+\sqrt{3}i}$ and $-\dfrac{a}{2}i$ respectively, where a is a positive real constant.

(i) Find the modulus and argument of p . [2]

(ii) Illustrate on an Argand diagram, the points P, Q and R representing the complex numbers p, q and $p+q$ respectively. State the shape of $OPRQ$.
Hence, find the argument of $p+q$ in terms of π and the modulus of $p+q$ in exact trigonometrical form. [6]

(iii) Find the smallest positive integer n such that $(p+q)^n$ is purely imaginary. [2]

4 **(a)** When studying a colony of bugs, a scientist found that the birth rate of the bugs is inversely proportional to its population and the death rate is proportional to its population. The population of the bugs (in thousands) at time t days after they were first observed is denoted by P. It was found that when the population is 2000, it remains constant.

(i) Assuming that P and t are continuous variables, show that $\frac{\mathrm{d}P}{\mathrm{d}t} = k\left(\frac{4}{P} - P\right)$, where k is a constant. [3]

(ii) Given that the initial population of the bugs was 4000, and that the population was decreasing at the rate of 3000 per day at that instant, find P in terms of t. [4]

(iii) Sketch the graph of P against t, giving the equation of any asymptote(s). State what happens to the population of the bugs in the long run. [2]

(b) Another population of bugs, N (in thousands) in time t days can be modelled by the differential equation $\frac{\mathrm{d}N}{\mathrm{d}t} = 4 + \frac{N}{t}$ for $t \geq 1$. Using the substitution $u = \frac{N}{t}$, solve this equation, given that the population was 1000 when $t = 1$. [3]

Section B: Statistics [60 marks]

5 The daily rainfall in a town follows a normal distribution with mean μ mm and standard deviation σ mm. Assume that the rainfall each day is independent of the rainfall on other days. It is given that there is a 10% chance that the rainfall on a randomly chosen day exceeds 9.8 mm, and there is a 10% chance that the mean daily rainfall in a randomly chosen 7-day week exceeds 8.2 mm.

(i) Show that $\sigma = 2.01$, correct to 2 decimal places. [4]

(ii) Find the maximum value of k such that there is a chance of at least 10% that the mean daily rainfall in a randomly chosen 30-day month exceeds k mm. Give your answer correct to 1 decimal place. [2]

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6 Miss Tan carried out an investigation on whether there is a correlation between the amount of time spent on social media and exam scores. The average amount of time spent per month on social media, x hours, and the final exam score, y marks, of 6 randomly selected students from HCI were recorded. The data is shown below.

x	80	84	70	74	58	48
y	44	40	49	45	58	82

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) It is found that the inclusion of a 7th point (x_7, y_7) will not affect the product moment correlation coefficient for the data. Find a possible point (x_7, y_7). [1]

Omit the 7th point (x_7, y_7) for the rest of this question.

(iii) State, with reason, which of the following equations, where a and b are constants, provides the most appropriate model for the relationship between x and y.

(A) $y = a + bx^2$,

(B) $e^y = ax^b$,

(C) $y = a + b\sqrt{x}$. [3]

(iv) Using the model chosen in part **(iii)**, estimate the score of a student who spent an average of 60 hours per month on social media, giving your answer correct to the nearest whole number. [2]

(v) Sam spends an average of 4 hours a day on social media. Assuming a 30-day month, suggest whether it is still reasonable to use the model in part **(iii)** to estimate his score. [1]

7 A cafe sells sandwiches in 2 sizes, "footlong" and "6-inch". The lengths in inches of "footlong" loaves have the distribution $\mathrm{N}(12.2, 0.04)$ and the lengths in inches of "6-inch" loaves have the distribution $\mathrm{N}(6.1, 0.02)$.

(i) Is a randomly chosen "footlong" loaf more likely to be less than 12 inches in length or a randomly chosen "6-inch" loaf more likely to be less than 6 inches in length? [2]

(ii) Find the probability that two randomly chosen "6-inch" loaves have total length more than one randomly chosen "footlong" loaf. [2]

Sue buys a "6-inch" sandwich 3 times a week.

(iii) Find the probability that Sue gets at most one sandwich that is less than 6 inches in length in a randomly chosen week. [2]

(iv) Given that Sue gets more than four sandwiches that are less than 6 inches in length in a randomly chosen 4-week period, find the probability that she gets exactly one such sandwich in the first week. [3]

8 The individual letters of the word PARALLEL are printed on identical cards and arranged in a straight line.

(a) Find the number of arrangements such that

(i) there are no restrictions, [1]

(ii) no L is next to any other L, [2]

(iii) the arrangements start and end with a consonant and all the vowels are together. [3]

(b) The cards are now placed in a bag and Tom draws the cards randomly from the bag one at a time.

(i) 4 cards are drawn without replacement. Find the probability that there is at least one vowel drawn. [2]

(ii) Tom decides to record the letter of the card drawn, on a piece of paper. If the letter on the card drawn is a vowel, Tom will put the drawn card back into the bag and continue with the next draw.

If the letter on the card drawn is a consonant, Tom will remove the card from subsequent draws. Find the probability that Tom records more consonants than vowels at the end of 3 draws. [3]

9 A company purchased a machine to pack shower gel into its bottles. The expected mean volume of shower gel in a bottle is 950 ml.

(a) The floor supervisor believes that the machine is packing less amount of shower gel than expected. A random sample of 80 bottles is taken and the data is as follows:

Volume of shower gel in a bottle (correct to nearest ml)	948	949	950	951	952	953	955
Number of bottles	9	22	36	6	4	1	2

(i) Find unbiased estimates of the population mean and variance, giving your answers correct to 2 decimal places. [2]

(ii) Write down the appropriate hypotheses to test the floor supervisor's belief. You should define any symbols used. [2]

(iii) Using the given data, find the p-value of the test. State what is meant by this p-value in the context of this question. [2]

(iv) It was concluded at $\alpha\%$ level of significance that the machine is indeed packing less amount of shower gel than expected. State the set of values of α. [1]

(b) Due to a change in marketing policy, the machine is being recalibrated to pack smaller bottles of shower gel with mean volume of 250 ml. The volume of a recalibrated bottle of shower gel is denoted by Y ml. A random sample of 50 bottles of y ml each is taken and the data obtained is summarised by:

$$\sum(y-250)=-25, \qquad \sum(y-250)^2=k.$$

Another test was conducted at the 1% significance level. The test concluded that the machine had been calibrated incorrectly. Find the range of values of k, correct to 1 decimal place. [4]

(c) Explain why there is no need for the floor supervisor to know anything about the population distribution of the volume of shower gel in a bottle for both parts **(a)** and **(b)**. [1]

10 In a game with a 4-sided fair die numbered 1 to 4 on each face, the score for a throw is the number on the bottom face of the die. A player gets to choose either option A or option B.

Option A: The player rolls the die once. The score x is the amount of money $\$x$ that the player wins.

Option B: The player rolls the die twice. The first score is x and the second score is y. If $y > x$, the player wins $\$2xy$, but if $y < x$, the player loses $\$(x - y)$. Otherwise, he neither wins nor loses any money.

(i) Find the expected amounts won by a player in one game when playing option A and when playing option B. Show that option B is a better option. [5]

(ii) Suggest why a risk averse player would still choose option A. [1]

(iii) Show that the variance of the amount won by a player in one game when playing option A is 1.25. [2]

In a competition, Abel and Benson each play the game 50 times. Abel chooses option A and Benson chooses option B.

It is given that the variance of the amount won by a player in one game when playing option B is $\frac{887}{16}$.

(iv) Find the distributions of the total amounts won by Abel and Benson respectively in the competition. [2]

(v) Show that the probability of the total amount won by Abel exceeding the total amount won by Benson in the competition is approximately 0.120. [3]