HWA CHONG INSTITUTION

2020 JC2 Preliminary Examination

Higher 2

9758/02

MATHEMATICS

14 September 2020

3 hours

Candidate Name	

CT Group	

Centre Number	**S**				

Index Number				

Write here how many additional pieces of writing paper you have used (if any).	

Candidates answer on the Question Paper.

Additional Materials: List of Formulae 26 (MF26)

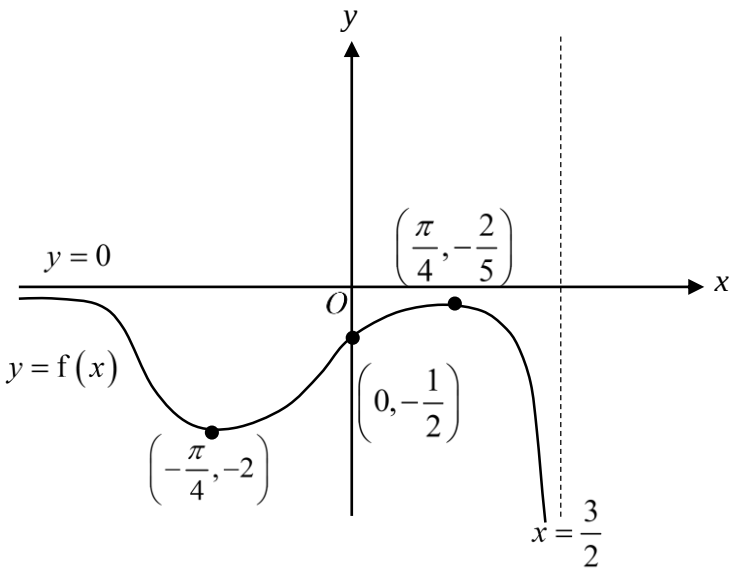
READ THESE INSTRUCTIONS FIRST

1. Write your name and class on this Cover Page and any additional writing paper you hand in.
2. Write in dark blue or black pen.
3. You may use an HB pencil for any diagrams or graphs.
4. **Do not use staples,** paper clips, highlighters, glue or **correction fluid.**
5. Answer **all** the questions and write your answers in the spaces provided in the Question Paper.
6. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
7. Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
8. Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
9. You are reminded of the need for clear presentation in your answers.
10. The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use			
Qn	**Marks**	**Total**	**Remarks**
1		6	
2		7	
3		8	
4		9	
5		10	
6		6	
7		7	
8		9	
9		11	
10		12	
11		15	
		100	

Section A: Pure Mathematics [40 marks]

1 The diagram below shows the graph of $y = \mathrm{f}(x)$ for $x < \frac{3}{2}$. The graph has stationary points at $\left(-\frac{\pi}{4}, -2\right)$ and $\left(\frac{\pi}{4}, -\frac{2}{5}\right)$, axial intercept at $\left(0, -\frac{1}{2}\right)$ and asymptotes $y = 0$ and $x = \frac{3}{2}$.



Sketch, on separate diagrams, the graphs of the following, showing clearly the coordinates of stationary points, axial intercepts and equations of asymptotes where applicable.

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

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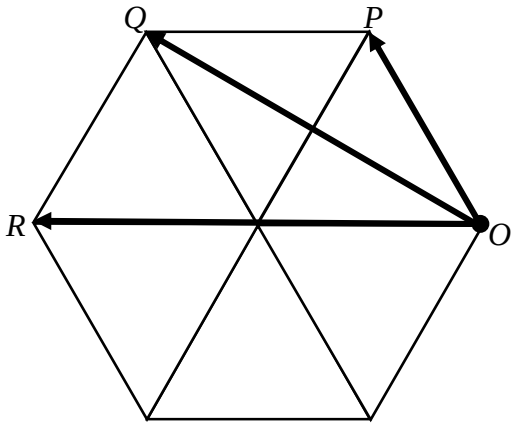
(ii) $y = \mathrm{f}'(x)$. [3]

2 **(a)** 6 equilateral triangles each with sides 5 units are arranged to form a regular hexagon as shown in the diagram below. By expressing $\overrightarrow{OQ}$ in terms of $\overrightarrow{OP}$ and $\overrightarrow{OR}$, or otherwise, find $\overrightarrow{OP}\bullet\left(\overrightarrow{OP}+\overrightarrow{OQ}+\overrightarrow{OR}\right)$. [3]



(b) $\overrightarrow{OE}, \overrightarrow{OF}$ and $\overrightarrow{OG}$ are non-zero position vectors relative to a fixed origin O for points E, F and G respectively. It is given that $2\overrightarrow{OE} - 5\overrightarrow{OF} + 3\overrightarrow{OG} = \mathbf{0}$. Show that E, F and G are collinear. State the ratio of $EF : EG$ and find $\overrightarrow{EF} \times \overrightarrow{EG}$.

[4]

3 **(i)** It is given that $\mathrm{f}(r) = (r-2)2^{r-1}$. Show that $\mathrm{f}(r+1) - \mathrm{f}(r) = 2^r ar$, where a is a constant to be determined. [2]

(ii) Hence find $\sum_{r=1}^{n} r2^r$. [3]

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(iii) Using the result in part **(ii)**, find $\sum_{r=3}^{n}(r+2)2^r$ in terms of n. [3]

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4 The variables x and y are related by the equation $y = x - \cos^{-1} x$, where $0 \leq x < 1$ and $0 < \cos^{-1} x \leq \frac{\pi}{2}$.

(i) Prove that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = x\left(\frac{\mathrm{d}y}{\mathrm{d}x} - 1\right)^3$. [4]

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(ii) Find the values of y, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at $x = 0$. [1]

(iii) By further differentiation, find the series expansion of y in ascending powers of x up to and including the term in x^3. [3]

(iv) Deduce the series expansion of $1+\frac{1}{\sqrt{1-x^2}}$ up to and including the term in x^2. [1]

5 The complex number a has modulus r and argument θ, where $r \in \mathbb{R}^+$ and $0 < \theta < \frac{\pi}{2}$. The complex number b is such that $b = -2ia$ and the complex number c is purely imaginary with negative imaginary part. It is given that $|b| = |c|$.

Let the points A, B, C and D represent the complex numbers a, b, c and $b + c$ respectively, and O be the origin.

(i) On a single Argand diagram, illustrate the points A, B, C and D, indicating clearly the modulus and argument of a, b and c. [4]

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(ii) State the angles $\angle AOB$ and $\angle BOC$ in radians. [2]

(iii) State the shape of the quadrilateral *OBDC*. Hence find, in terms of r and θ, the complex number $b+c$ in exponential form. [4]

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Section B: Statistics [60 marks]

6 **(a)** Given that events A and B are independent, show that A' and B are also independent. [3]

(b) A, B and C are events such that events A and B are independent and events A and C are mutually exclusive. It is given that

$\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = b$, $\mathrm{P}(B \cap C) = 0.15$ and $\mathrm{P}(A' \cap B' \cap C') = 0.2$.

With the help of a Venn diagram, find the range of values for $\mathrm{P}(A \cap B)$. [3]

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7 A team of 15 students was selected for an outdoor education trip. One student volunteered to be the trip leader while another volunteered as the assistant trip leader. They decided to have some ice-breaker games, where all 15 students sat in a circle.

(a) Find the probability that both leaders were not seated together. [2]

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During the ice-breaker games, it was realised that 3 of the other 13 students belonged to the same class.

(b) Find the probability that none of these three students sat next to each other, given that both leaders were not seated together. [3]

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7 [Continued]

(c) After the ice-breaker games, both leaders decided to break the team up into two groups to discuss about administrative matters and training program for the trip. The group which discussed about the training program consisted of 7 students. Find the probability that the trip leader was heading the training program group while the assistant leader was in charge of the administrative matters group.

[2]

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8 A set of 9 cards are numbered 1, 1, 2, 3, 3, 4, 5, 5 and 6. A round of matching game is played where two cards are drawn, one at a time without replacement. A match occurs when the two cards drawn have the same number.

(i) Given that the sum of the numbers on the two cards is less than 5, find the probability of a match occurring in a game. Hence, or otherwise, determine if having a match is independent of having a sum of the numbers on the cards less than 5 for a game. [3]

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8 [Continued]

(ii) Dave decides to try his luck at this game repeatedly until he obtains a match. What is the least number of rounds he will have to attempt such that the probability of obtaining a match is at least 75%? [4]

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(iii) Instead of drawing only two cards in a round, Dave continues to draw the cards until he obtains a match. He draws one card at a time without replacement. Find the probability that a match occurs on his third draw. [2]

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9 A hand sanitiser company, Cleanser, claims that the mean volume of each bottle of their hand sanitiser is 15 ml. The consumer association took a random sample of 80 bottles and measured the volume of each bottle. It was found that the sample mean and the sample variance were $\bar{x}$ and 2 ml^2 respectively. A test was conducted at the 5% level of significance to test if Cleanser has overstated the mean volume of its hand sanitiser.

(i) Explain, in the context of the question, the meaning of a random sample. [1]

(ii) Explain, in the context of the question, the meaning of 'at the 5% level of significance'. [1]

(iii) Find the unbiased estimate of the population variance. [1]

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It was found that there was insufficient evidence to conclude that Cleanser had overstated the mean volume of their hand sanitiser.

(iv) By defining any parameters used and stating the hypotheses clearly, find the range of values of $\bar{x}$, correct to 2 decimal places. [5]

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9 [Continued]

Another random sample of 80 bottles of hand sanitiser was taken from another hand sanitiser company, Germsfree. The distribution of the volume of the 80 bottles of Germsfree sanitisers were given in the table below.

Volume (ml)	13	14	15	16	17
Number of bottles	14	17	24	12	13

(v) Find an unbiased estimate for the population mean and variance of volume of a bottle of Germsfree sanitiser. [2]

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(vi) The same test was performed on the sample from Germsfree to see if Germsfree had also overstated the mean volume of its hand sanitiser.

It was found that both samples from Cleanser and Germsfree had the same mean. Let p_1 and p_2 be the $p-$value from the test on the sample from Cleanser and Germsfree respectively. Without doing any further calculation, state with justification an inequality relating p_1 and p_2. [1]

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10 Pat is invited to a game show. In the game, she is presented with a box with k balls with either 1, 2, 3 or 4 printed on each of the balls. The probability of drawing a ball with number i printed on it is $\frac{i}{k}$, where $i = 1, 2, 3, 4$. Two balls are drawn one after another with replacement. The prize money that Pat gets is $\$W$, where W is the mean of the two numbers printed on the balls drawn.

(i) Show that $k = 10$. [1]

(ii) Find the probability distribution of W. [4]

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(iii) If T is an estimator of a population parameter θ and $\mathrm{E}(T) = \theta$, then T is called an unbiased estimator of θ. Show that W is an unbiased estimator of the mean of all the numbers printed on the balls in the box. [1]

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10 [Continued]

(iv) For the first round Pat will play the game 5 times. She can proceed to the second round if she earns more than \$2.50 for at least 3 games in the first round. Find the probability that Pat will proceed to the second round. [3]

(v) Pat proceeds to the second round. She will draw one ball from another box containing a total of m identical black or white balls, of which 2 are black. If Pat draws a black ball, she will receive $\$3m$. If she draws a white ball, she will lose $\$\left(\frac{m}{2}\right)$. Given that there is at least one white ball in the box, find the range of values of m such that the expected amount earned in the second round for Pat is non-negative. [3]

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11 In a Junior College, the Physical Education (PE) department is analyzing the running time taken by girls from 2 cohorts, C1 and C2, to complete the 2.4 km running test.

The running time in minutes of a randomly chosen C1 girl to complete the 2.4 km running test follows a normal distribution with mean 14.8 minutes and variance 2 minutes^2.

(i) A group of k C1 girls is selected at random. Given that there is more than 99% chance that their average running timing for the 2.4 km running test is not more than 15 minutes 40 seconds, find the least value of k. [3]

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The PE department adopts a new training strategy on the C1 girls. After going through the new training strategy, the running time, in minutes, of a randomly chosen C1 girl to complete the 2.4 km running test is now said to have the distribution $\mathrm{N}\left(\mu, \sigma^2\right)$.

The girls will achieve at least a B grade if they are able to complete the 2.4 km running test in no more than 15 minutes 40 seconds. The passing timing for the 2.4 km running test for girls is 17 minutes 20 seconds.

(ii) Given that there is a 70% chance of a randomly chosen C1 girl attaining at least a B grade and a 5% chance of her failing the 2.4 km running test, find the values of μ and σ. [3]

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11 [Continued]

(iii) The running time in minutes of a randomly chosen girl from C2 to complete the 2.4 km running test follows a normal distribution with mean 15.1 minutes and variance 14 minutes^2.

The department realised that there is an error in the given distribution of the running time of C2 girls to complete the 2.4 km running test. Explain with justification why this given distribution is incorrect. [1]

Assume the running time in minutes of a randomly chosen girl from C1 to complete the 2.4 km running test still follows a normal distribution with mean 14.8 minutes and variance 2 minutes^2. After correction, the running time in minutes of a randomly chosen girl from C2 to complete the 2.4 km running test is now said to have the distribution $\mathrm{N}(15.1, 1.4)$.

(iv) What is the probability that the total running time taken by 4 randomly chosen C1 girls to complete the test differs from four times the running time taken by a randomly chosen C2 girl to complete the 2.4 km running test by less than a minute? [3]

(v) **(a)** State two assumptions for the number of girls in a C2 class of n girls failing the 2.4 km running test to be well modelled by a Binomial distribution. [2]

(b) The number of girls in a C2 class of n girls failing the 2.4 km running test is now assumed to be well modelled by a Binomial distribution. Given that the probability of less than 3 girls from a C2 class of n girls failing the 2.4 km running test in the following year is 0.977, find the value of n. [3]

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